INSTRUCTIONS

FOR THE USE OF

A. W. FABER'S

IMPROVED

CALCULATING RULE.

BY

CHARLES N. PICKWORTH, Wh. Sc.,

Author of "The Slide Rule," etc.

MANUFACTORY ESTABLISHED 1761.

Prize Medals from all Important

International and Industrial Exhibitions.

MANUFACTORIES IN

GERMANY, FRANCE, AND UNITED STATES.

HOUSES IN

LONDON, PARIS, BERLIN, AND NEWARK, N.J., U.S.A.

A. W. FABER,

41–47, DICKERSON STREET AND 64–88, HECKER STREET,

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PRINTED IN ENGLAND.

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INSTRUCTIONS

FOR THE USE OF

A. W. FABER'S **Improved Calculating Rule.**

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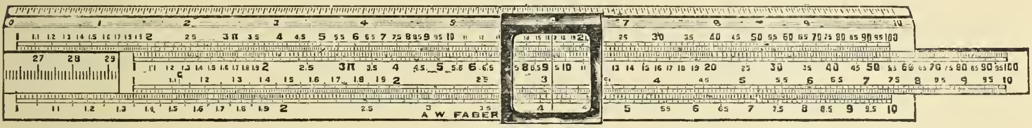

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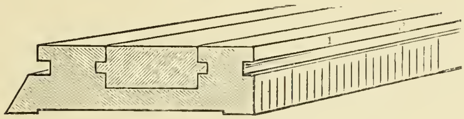

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Introduction.

It is a somewhat surprising fact that in an age in which such an immense amount of attention is given to labour-saving tools and appliances generally, so little advantage has been taken of the Calculating Rule—an instrument which offers a ready means for mechanically performing all the varied calculations which are required by the engineer or architect, the chemist or the power user.

It must be admitted, however, that latterly the merits of the Calculating Rule have become more generally recognised and its advantages more fully understood; hence there is little doubt that the instrument will become much more popular in the future than it has been in the past.

The great improvements effected in the Calculating Rule by the addition of the cursor, as well as by the adoption of a systematic arrangement of the various scales, have done much to encourage the use of the instrument, but a concise and practical handbook explaining its many and varied uses is indispensable to the fullest appreciation of the merits of this useful instrument. This the writer has endeavoured to supply.

The Construction of the Calculating Rule.

The A. W. FABER Calculating Rule consists of a "stock" or "body" about 11 inches in length, $1\frac{1}{4}$ inches in width and $\frac{3}{8}$ inch in thickness. In this stock a groove is formed which carries a moveable strip or "slide," the upper surface of which is level with that of the

stock of the rule. Upon the " face " of the rule thus formed, four scales are engraved, one near the upper edge of the slide being adjacent to a precisely similar one on the upper part of the stock: while another near the lower edge of the slide has adjacent to it a precisely similar scale on the lower part of the stock of the rule.

In close proximity with the face of the rule, the cursor is arranged to slide freely backwards and forwards. In some instruments the cursor consists of a metal strip, bridging over the several scales, and provided with chisel-like projections which lie over the scales, and the edges of which may be brought to any division line of the scales as required.

In the A. W. FABER Rule a form of cursor is used which is much to be preferred to the foregoing. This consists of a light metal frame, sliding as before in small grooves formed in the edges of the stock of the rule, and containing a piece of glass about 1 inch square, in the centre of which is a fine black line, which can thus be brought over any division line. One great advantage of this arrangement is that it leaves visible the adjacent graduations of the several scales, thus enabling the operator to readily assign the correct value to the reading sought.

In several other respects this form of cursor will be found greatly superior to the more usual form, especially noticeable being the facility with which the settings may be made in complicated calculations.

The A. W. FABER Calculating Rule is made of well-seasoned boxwood, provided with celluloid facings upon which the several scales of the instrument are engraved. The dull white surface of the celluloid with the division lines in blue give very distinct reading without fatiguing the eyes.

The only objection to the celluloid-faced rule is, that this facing material is somewhat unreliable under variations of atmospheric conditions. However, as the graduations of the Calculating Rule are in every case *comparative* and not *absolute*, the expansion and contraction cannot affect the accuracy of the instrument.

The Scales.

Reference has already been made to the fact that the face of the Calculating Rule has four scales engraved thereon. The upper one of these, found on the stock of the rule, is usually designated the " A " scale. Immediately adjacent to it is a precisely similar scale

on the *slide*, and known as the "B" scale. On the lower edge of the slide is found the "C" scale, and finally, the precisely similar scale on the lower part of the stock is the "D" scale.

As facility in the use of the Calculating Rule depends in very large measure upon the readiness with which the operator can assign the correct value to each graduation of these several scales, it will be necessary to explain, at some length, the notation and sequence of the several sets of graduations referred to as the "scales."

Taking first the A scale (which as explained is exactly similar in every respect to the B scale), it will be seen that this commences with 1 at the left-hand upper corner of the rule. This is known as the left-hand "index" of the scale. Following the scale along, it will first be noticed that it consists of two similar sets of divisions—the primary divisions of the first half being numbered from 1 to 10, while those of the second or right-hand half of the scale are numbered from 10 to 100.

It will be further noticed that the spaces comprised between the successive primary divisions are of constantly diminishing length. This is in consequence of the fact that the position of each of the several division lines is such that it represents, by its distance from the left-hand index, the *logarithm* of the number by which it is distinguished. Thus if the whole length 1 to 10 be imagined divided into 1000 equal parts, the graduation marked 2 would be placed at the 301st division, since the logarithm of 2 is ·301. Similarly division line 3 would be placed at the 477th, and line 4 at the 602nd division of the imaginary evenly-divided scale, and so on, the relative distances of the several primary graduations from the index being proportional to the logarithms of the numbers by which they are distinguished.

The same principle determines the positions of the other divisions of this and also of the other scales, and applies equally to the several sub-divisions of all.

It is well known that the addition of the logarithms of two numbers gives the logarithm of their product, while the subtraction of the logarithm of a smaller number from that of a greater gives the logarithm of the result of dividing the greater number by the lesser.

With the Calculating Rule such lengths of the scales representing various quantities may be readily added to or subtracted from each other by means of the slide. A simple example will best illustrate the method. Thus let it be required to find the result of multiplying 2 by 2. The logarithm of 2 we have first seen is ·301. Adding this

to ·301 we obtain ·602, or the logarithm of 4. With the slide rule we proceed to effect this addition mechanically by drawing the slide to the right until the left-hand index (1) of scale B agrees with 2 on scale A. Then over 2 on B is found 4 on A. With the slide so set, it is readily perceived that to a length 1—2 on A, there has been added a length 1—2 on B, the result of which addition is at once read off on A over 2 on B. The same setting of the rule shows that in dividing 4 by 2, we take from a length 1—4 (representing 4) on A, a length 1—2 (representing 2) on B, the result (2) being read off on A over the left-hand index on B.

It is upon this principle that all the operations effected by the Calculating Rule depend, but beyond this rudimentary explanation, no further inquiry into the properties of logarithms need here be entered into.

Reverting again to the several divisions of the upper scales, it will be observed that all the primary spaces as 1—2, 2—3, 3—4 to 90—100 are each divided into ten smaller spaces, but since the lengths of the successive primary spaces rapidly diminish, it becomes impossible to sub-divide each of these to the same extent as is possible with 1—2 or 10—20.

Commencing with the left-hand index (1) of A, and assigning to that division line a value of 1, it will be seen that the first small sub-division met with represents 1·02, the second 1·04, and so on until the first main sub-division line marked 1·1 is reached. The same method is pursued until the primary division line 2 is reached. The succeeding primary spaces 2—3, 3—4, 4—5 are each divided into ten parts, each of which is again subdivided into two parts. Thus in this part of the scale, the space from each successive division line to the next represents an addition of ·05, so that commencing at 2, we should read 2, 2·05, 2·10, 2·15, 2·20, etc., to 4·95 and 5·00. Regarding the primary spaces 5—6, 6—7, 7—8, 8—9 and 9—10, the rapidly diminishing lengths of these only permit them to be each divided into tenths, so that from 5 the reading is:—5·1, 5·2, 5·3, etc., to 9·9 and 10·0.

Precisely the same plan of sub-division is adopted in the right-hand half of the upper scales, but in this case every division line is of tenfold value as compared with those just enumerated. Although more minute sub-division of the scales is not possible with this length of rule, it becomes an easy matter to estimate by the eye the further sub-division of the several spaces. With a little practice expertness is soon acquired in correctly reading off any result which falls intermediately of the engraved division lines.

It is highly important that the operator should be able to read the scales with facility, and in this connection he will find Fig. 1 in the accompanying sheets of illustrations of material assistance. In this, the position of the following values are clearly indicated:—

1·26	1·58	1·77	1·83	1·975	2·20	2·29	2·65
3·74	4·82	4·96	5·59	6·42	7·58	8·65	9·82
11·2	14·4	16·9	18·3	19·75	23·0	27·2	38·7
46·5	48·4	52·1	59·4	67·5	81·7	94·9	

Coming now to the lower scales C and D (of which we need only consider the latter, since they are exactly similar), it will be seen that in this case the scale extends from 1 (left-hand index) to 10 (right-hand index). It resembles, therefore, the left-hand half of the A scale, but as it is of twice the length of that scale, the sub-divisions are more numerous and the reading therefore more minute. Commencing at the left-hand index (1) of D, and assigning thereto a value of 1, the first sub-division met with would be read 1·01, the second 1·02, and so on until the first numbered division 1·1 is met, and continuing in this way until the primary graduation 2 is reached. The spaces 2—3 and 3—4 are each divided into 50 parts, so that the values assigned to these will be 2·02, 2·04, etc., to 3·98 and 4·00. The remainder of the primary spaces of this scale are divided into 20 parts, so that we read 4·05, 4·10, 4·15, etc., to 9·95 and 10. Fig. 2 will assist in rendering this clear, the following values being marked thereon:—

1·25	1·85	2·35	2·58	3·49	4·57	9·34

From the foregoing explanation it will be seen that either the upper scales A and B, or the lower pair C and D may be used for ordinary multiplication, division, proportion, etc. The use of the upper scales has the advantage of enabling a much larger range of values to be read off without a second movement of the slide being necessary. On the other hand, the scales being of only one-half the length of C and D, more accurate results can be read on the latter. Generally, therefore, it may be said that when only approximate results are sought, the upper scales are to be preferred, but when greater exactitude is needed, the lower pair should be employed.

It is important to observe that, when using either pair of scales, the values assigned to the several divisions of the scales depend directly upon the value assigned to the left-hand index figure of the scale. But in such calculations as involve the use of the upper and lower scales in conjunction, the value assigned to the left index figure (1) of the D scale determines the value of all the divisions on the rule.

It must further be clearly understood that although this index figure is designated 1, it may be taken equally to represent any value that is a multiple or submultiple of 10. Thus this left hand index (1) of the D scale may be regarded as 1, 10, 100, 1000, etc., or as 0·1, 0·01, 0·001, 0·0001, etc. ; but once the initial value is assigned to the index, the ratio of value must be maintained throughout the whole scale. For example, if 1 on D is taken to represent 10, the main divisions 2, 3, 4, etc., will be read as 20, 30, 40, etc. On the other hand, if the fourth main division is read as 0·004, then the left index figure of the scale will be read as 0·001. The figured sub-divisions of the main space 1—2 are to be read as 11, 12, 13, 14, 15, 16, 17, 18 and 19 if the index represents 10, and as corresponding multiples for any other value of the index.

Independently considered, these remarks apply equally to the A or B scale, but in this case the notation is continued through the second half of the scale, the figures of which are read as tenfold values of the corresponding figures in the first half of the scale. Thus, if the left index of A is read as 10, the scale will run 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 (centre index), 200, 300, 400, etc., to 1000 at the right-hand index. The reading of the intermediate divisions will of course, be determined by the values assigned to the primary divisions.

Before proceeding to illustrate the methods of using the Calculating Rule, it is desirable to indicate the mathematical relation which exists between the upper and lower scales.

It has been pointed out that each of the two similar scales, which together form the A scale line, is only one-half the length of the D scale. Therefore, starting at the left-hand index, the actual length representing the logarithm of 2 on the D scale would be twice that representing the logarithm of 2 on the A scale, and so on throughout the whole length of the scale. In other words, the lengths representing the logarithms of the numbers on the A scale are, considered comparatively, equal to those on the D scale multiplied by 2, and therefore, from the well-known properties of logarithms, equal to the logarithms of the *squares* of the numbers on the latter scale. Since the numbers and not the logarithms are engraved on the rule, it consequently results that opposite any number on D, we find its *square* on A, and, conversely, that under any number on A is its *square* root on D, this relationship holding for every division on the rule. The cursor fitted to the rule enables the coinciding divisions of the upper and lower scales to be accurately determined, while, as

will be shown later, it also greatly facilitates operations generally, especially when of a complex character.

On the reverse side of the slide three scales are engraved. To these reference will be made later, when dealing with the trigonometrical applications of the Calculating Rule. It is now only necessary to further note that on one edge of the instrument a scale of inches is provided, while on the other edge is a millimetre scale. This latter, which runs up to 260 millimetres on the edge of the rule, may be continued to 510 millimetres by means of the supplementary scale seen at the bottom of the groove in the rule when the slide is withdrawn. Thus, if a length of 420 millimetres be required, the slide is withdrawn to the right until the left-hand end of it agrees with 42 on the scale inside the groove. The total length from the left-hand end of the stock of the rule to the right-hand extremity of the slide is then 420 millimetres.

Finally, on the back of the rule will be found a number of gauge points and other information of special service to engineers and technical men generally.

Proportion.

One of the most simple, but at the same time one of the most important applications of the Calculating Rule, is its use for the solution of problems involving the proportion, or ratio of quantities. As before explained, either the upper or lower pair of scales may be used, depending upon the degree of accuracy required. Assuming the C and D scales to be employed, it will be seen that, with the left-hand indices of C and D placed in coincidence, the ratio existing between all corresponding divisions of these scales is unity (1). If, however, the slide be moved to the right, so that the index of C falls opposite 2 on D, it will be seen that the numbers on D will bear to the coinciding numbers on C a ratio of 2 to 1. Similarly a definite relationship exists between the numbers on these two scales wherever the slide may be placed. Hence the rule for proportion which is apparent from the foregoing, may be expressed as follows :—

RULE FOR PROPORTION.—*Set the first term of a proportion on the* C *scale to the second term on the* D *scale, and opposite the third term on the* C *scale read the fourth term on the* D *scale.*

EXAMPLE 1. What is the fourth term in the proportion $15 : 35 :: 19{\cdot}5 : x$?

Draw the slide to the right until 15 on C coincides with 35 on D, and under 19·5 on C read 45·5 on D, the required fourth term.

It will be noticed that the graduations used in the above example are marked 1·5, 3·5, 1·95 and 4·55 respectively, but as previously explained these engraved values are also to be taken as representing any multiple value thereof, care being taken to give the corresponding multiple value to the result. In this case the engraved figures are increased tenfold; therefore the resulting fourth term is also increased tenfold, being read as 45·5.

This method may be readily modified so as to give from *any three* known terms of a proportion the remaining term of the expression. Thus, given the 2nd, 3rd and 4th terms of the above example, the method of procedure would be:—Set the 3rd term (19·5) on C to the 4th term (45·5) on D, and opposite the 2nd term (35) on D read the required 1st term (15) on C. All it is necessary to observe is that the 1st and 3rd terms are always found on one scale, and the 2nd and 4th on the other.

It should be noted that as a consequence of the constant proportion existing between the numbers on the two scales, the fraction $\frac{\text{1st term}}{\text{2nd term}}$ may be expressed as $\frac{1}{x}$ or as its reciprocal $\frac{x}{1}$, the value of x in the first case being read on the D scale under the index of the slide, and in the second case on the C scale over the index of the D scale. Thus, in reducing vulgar fractions to decimals, the decimal equivalent of $\frac{3}{16}$, for example, is determined by placing 3 on C to 16 on D, when over the index or 1 of D we read 0·1875 on C. In this case the terms are $3 : 16 : : x : 1$.

For the inverse operation—to find a vulgar fraction equivalent to a given decimal—the given decimal fraction on C is set to the index of D, and then opposite any denominator on D is found the corresponding numerator of the fraction on C.

If the index of C be placed coincidentally with 3·1416 on D, it will be clear from what has just been said that this ratio exists throughout between the numbers of the two scales. Therefore against any *diameter* of a circle on C will be found the corresponding *circumference* on D. As the graduations of the rule, however, do not admit of a very exact setting being made in this way, it has become customary under this and similar circumstances, to set the slide by some more convenient, but still sufficiently exact, equivalent ratio. For example, instead of the value of π just given, the ratio $\frac{22}{7}$ is frequently adopted.

In the A. W. FABER Calculating Rule a special graduation line will be found on the A and B scales, marking the exact position of π. Therefore all that is necessary is to draw the slide to the right until 1 on B is exactly under the line marked π on A. With the rule thus set, the diameters of any circles taken on B will have immediately above them their corresponding circumferences on A.

The ready solution which the Calculating Rule affords of all problems involving ratio or proportion, renders the instrument of peculiar value for determining the equivalent values in different denominations of the various weights, measures, etc.

EXAMPLE 2. Convert centimetres into inches.

As 1 inch is equivalent to 2·54 centimetres, all that is necessary is to place 1 on B to 2·54 on A, as shown in Fig. 3, when above any number of inches on B will be found the equivalent length in centimetres on A, or *vice versa*. In the figure the following conversions are noted :—

1·25 inches	= 3·18 centimetres.	2·75 inches	= 6·99 centimetres.
2·50 „	= 6·35 „	3·50 „	= 8·89 „

In the above example the left-hand index of both the A and B scales have a value of 1. Frequently, however, the two scales used in conversions of this kind differ considerably in value, this of course depending upon the value of the two factors which determines the ratio existing between the two denominations. Thus, in converting English miles to metres, or *vice versa*, the left-hand index of B representing 1 mile would be set to 1610 on A, representing the equivalent number of metres. In making this setting, 1 on B is most conveniently set to 1·61 on A, since this enables more of the rule to be used than if the graduation 16·1 is employed. The fact must, however, be borne in mind that while the B scale is read as engraved, all the divisions of the A scale are to be read as of a thousandfold value.

When the C and D scales are used for such conversions, a portion of one scale will project beyond the other. To read this portion of the scale the runner is brought to whichever index (1 or 10) of the C scale, which falls within the rule. The slide is then carefully moved (without disturbing the cursor) until the other index of the C scale coincides with the runner. The remainder of the equivalent ratios may then be read off. It must be remembered that if the slide is moved in the direction of notation—that is, to the right, the values read thereon have a tenfold greater value; while if the slide is moved to the left, the readings thereon are decreased in a tenfold degree.

Inverse Proportion.—In such problems as those in which "more" requires "less" or "less" requires "more," the case is one of *inverse proportion,* and although from what has been said it will be seen that this form of proportion is quite readily dealt with by the preceding method, it may be observed that the working is simplified to some extent by inverting the slide so that the C scale is adjacent to the A scale. By the aid of the cursor the values on the inverted C (written C^1) scale and on the D scale can be read off. These will now constitute a series of inverse ratios. Thus in the proportion 4 : 3 : : 8 : 1·5, the 4 on the C^1 scale is brought opposite 3 on D, when under 8 on C^1 is found 1·5 on D.

Multiplication.

Multiplication, which is only a form of proportion in which one of the terms is 1, has next to be considered. The rule for this operation, which will be apparent from the foregoing, is:—

RULE FOR MULTIPLICATION.—*Set the index of the* B *scale to one of the factors on the* A *scale, and over the other factor on* B *read the product on* A *; or, set the index of* C *to one factor on* D, *and under the other factor on* C *read the product on* D.

EXAMPLE 3. Multiply 3·5 by 1·3.

Using the upper pair of scales, 1 on B is placed under 3·5 on A, and over 1·3 on B we read 4·55 on A, as shown in Fig. 4.

EXAMPLE 4. Multiply 135 by 38.

Using the lower pair of scales, and placing 1 on C to 1·35 (read as 135) on D, we read under 3·8 (read as 38) on C the required product 5·13 (read as 5130) on D.

This example indicates the necessity for taking care to give the correct value to the result. Rules might be given for locating the position of the decimal point, but these are cumbersome and not easily remembered.

Generally, it may be said that the number of digits in a product is obvious from the value of the factors, but if any doubt exists, a rough mental calculation can be readily made. Practice soon enables the operator to read off the correct result without hesitation.

In general it will be seen that the extent to which the C and D scales are sub-divided is such as not to enable more than three figures in either factor to be dealt with. For the same reason it is impossible to directly read more than the first three figures of any product, although it is often possible, by mentally dividing the smallest space

involved in the reading, to correctly determine the fourth figure of a product. Necessarily this method is only reliable when used in the earlier parts of the C and D scales. However, the last figure of a three-figure, and in some cases the last of a four-figure product can be readily ascertained by an inspection of the factors.

Thus, in such a case as $1{\cdot}69 \times 7 = 11{\cdot}83$, we place 1 on B to 1·69 on A, and find the result on A over 7 on B. This is seen to be more than 11·8 and less than 11·9, but as it is evident that $7 \times 9 = 63$, it is clear that the final figure is 3 and the correct product is therefore 11·83.

In cases where more than three figures enter into either or both of the factors, the fourth and following figures must be neglected. It should be noted that if the first neglected figure is 5, or greater than 5, it will generally be advisable to increase by 1 the third figure of the factor employed. Generally it will suffice to make this increase in only one of the factors; but it is obvious that in some cases greater accuracy will be obtained by increasing both factors in this way.

The continued multiplication of a number of factors is, by the aid of the cursor, very conveniently effected, this being one of the marked advantages over the older forms, of the type of rule now under consideration. With the former it is necessary to read off the result of each separate multiplication, the product so found being used as one of the factors in the next operation, this process being repeated until all the factors have been taken into account. With the cursor, however, the operation is continuous, so that not only is time saved but much greater accuracy secured, since the errors incidental to the several separate settings are altogether avoided.

Example 5. Find the product of $3{\cdot}2 \times 75 \times 4{\cdot}2 \times 1{\cdot}5 = 1512$.

Using the lower scales, we place the right-hand index of C to 3·2 on D, and bring the cursor so that the line thereon is over 75 on C. Bringing the right-hand index of C to the cursor line, the latter to 4·2 on C, and the left-hand index of C to the cursor, we read under 1·5 on C the required product 1512 on D, the last figure being estimated by the eye.

Division.

As division is merely the inverse of multiplication, we obtain the following:—

Rule for Division.—*Set the divisor on* B *under the dividend on* A, *and read the quotient on* A *over the index of* B; *or, set the divisor on* C *over the dividend on* D, *and read the quotient on* D *under the index of* C.

EXAMPLE 6. Divide 4·55 by 3·5.

Using the upper scales, place 3·5 on B under 4·55 on A, in the manner shown in Fig. 5. Then over 1 on B is read 1·3 on A.

EXAMPLE 7. Divide 76·5 by 4·5.

Here, 4·5 on B is placed to 76·5 on A, as shown in Fig. 6, when over 1 on B is read 17 on A, the required quotient.

Combined Multiplication and Division.

The combination of the rules for multiplication and division as given in the foregoing is often of the greatest possible service, especially in solving problems which may be represented by the equation :—

$$\frac{a \times b}{c} = x$$

In such a case, the division of a by c is first effected by placing c on C to a on D. Then, without paying any attention to the result of this operation, the multiplication of the quotient by b is effected by reading under b on C the required value of x on D.

EXAMPLE 8. $\frac{30 \times 28{\cdot}9}{51} = 17$

Setting 51 on C to 30 on D, we read 17 on D, under 28·9 on C.

More complicated cases are such as are represented by :—

$$\frac{a \times b \times c \times d}{e \times f \times g \times h} = x$$

It is in examples of this kind that the advantage of the cursor becomes strikingly apparent, since by its aid the result of such a lengthy calculation may be readily found without any regard being paid to the intermediate values which result from the successive steps in the operation.

EXAMPLE 9. $\frac{35 \times 71 \times 17 \times 42{\cdot}8}{42 \times 20 \times 5{\cdot}8 \times 8{\cdot}5} = 43{\cdot}6$

The operations necessary to solve this problem are as follows :— Set 42 on C to 35 on D, and bring the cursor to 71 on C. Bring 20 on C to the cursor, the runner to 17 on C, 58 on C to cursor, the cursor to 42·8 on C, and 8·5 on C to cursor. Then under the right-hand index of C read 43·6 on D.

The position of the decimal point in the answer in such examples as these is best determined by a rough cancellation, which in most cases can be effected mentally.

It will often be found that when the slide has been finally set the result cannot be read off, for the reason that it lies beyond the extremity of the rule, either to the right or left. In such a case, the cursor is brought to whichever index of C is available and the slide moved until the other index coincides with the cursor, when the result can be read off.

In complex cases of either continued multiplication and division combined, some discrimination should be used as to the sequence in which the various factors are taken into account. It will be frequently found that by taking the factors in a different order from that in which they are presented in the original problem, it is possible to avoid changing the position of the slide as would otherwise be necessary, owing to the reading falling outside the limits of the scale. Obviously, however, it is impossible to give rules for procedure in such cases, as facility in dealing with the factors in the most advantageous manner can only be acquired by practice.

The Inverted Slide.

By inverting the slide, as previously explained, so that the C scale lies in contact with the A scale, and setting the slide with the right and left-hand indices coinciding, it will be found that the result of multiplying any number on D by the coincident number on the inverted scale of C (C^1), readily effected by the cursor, is in all cases equal to 10.

As a consequence of this, if we read the numbers on C^1 as decimals, they form the *reciprocals* of those coinciding with them on D, and *vice versa*. Thus 2 on D is found opposite 0·5 on C^1; 3 on D opposite to 0·333; while opposite 8 on C^1 is 0·125 on D, etc.

It will now be perceived that if we attempt to apply the ordinary rule for multiplication, with the slide inverted, we shall actually be multiplying the one factor taken on D by the *reciprocal* of the other taken on C^1. But multiplying by the reciprocal of a number is equivalent to dividing by that number; while *dividing* a factor by the reciprocal of a number is equivalent to multiplying by that number. Hence it follows, that with the slide inverted the operations of multiplication and division are reversed. In multiplying, therefore, with the slide inverted we place (by the aid of the cursor) one factor on C^1, opposite the factor on D, and read the result on D under either index of C^1. It results that with the slide thus set, any pair of coinciding factors on C^1 and D will give the same constant product found on D under the index of C^1.

Squares and Square Roots.

It has been previously explained that the relationship which exists between the upper and lower scales is such that immediately over any number on D will be found its *square* on A. Similarly over any number on C will be found its square on B.

Conversely it follows that under any number on A is found its *square root* on D, and under any number on B is its square root on C. We have therefore the following:—

RULE FOR SQUARING A NUMBER.—*Set the cursor to the given number on* D, *and read off the required square on* A *under the cursor.*

In place of the cursor the indices of the B and C scales may be employed, if desired, as shown in Fig. 7.

EXAMPLE 10. Find the square of 93.

Setting the cursor to 27 on D, the square is read on A as 729. the true result being 8649.

EXAMPLE 11. Find the square of 27.

Setting the cursor to 27 on D, the square is read on A as 729.

It will be observed that in Example 10 the result is read on the *right-hand* scale of A, while in Example 11 it is read on the *left-hand* scale of A.

In the first case the result is seen to contain *twice* the number of figures that the original number contained, while in the second case, the number in the result is one less than twice that number. This brings us to the:—

RULE FOR NUMBER OF FIGURES IN THE SQUARE OF A NUMBER. *If the result is read on the right-hand scale of* A, *the number of digits is equal to twice the number of digits in the original number; but if the result is read on the left-hand scale of* A, *the number of digits is one less than twice the number in the original number.*

In finding the squares of decimals care must be taken in estimating the number of digits in the number and in the result. Thus:—

EXAMPLE 12. Find the square of ·00405.

Placing the cursor to 405 on D, the reading on A is found to be 164 [16405]; but in order to correctly state the result it must be observed that the number of digits in the original number is —2, since two cyphers follow the decimal point. Therefore by the rule just given the number of digits in the result (since the reading is found on the right-hand scale of A) is $-2 \times 2 = -4$. Therefore the result is written ·0000164.

Example 13. Find the square of ·0155.

Here the result, read as 24, will be found on the left-hand scale of A, and the number of digits it contains will therefore be $(-1 \times 2) - 1$ or -3, so that the result will be read as ·00024 [·00024025].

Square Roots.—After what has been said with reference to squaring a number, the procedure in the inverse operation of extracting the square root will be evident, since it is only necessary to set the runner to the given number on A and to read off the required square root on D under the cursor.

Example 14. Find the square root of 45·5.

Set the cursor (or the indices of B and C as shown in Fig. 7) to 45·5 on A, and read 6·74 ([6·745 +] on D. This operation is shown in Fig. 7.

It must be observed that if the number consists of an *odd* number of digits, it is to be taken on the *left-hand* portion of the A scale, and the number of digits in the root is $\frac{N+1}{2}$, N being the number of digits in the original number. When, however, there is an *even* number of digits in the given number, it is to be taken on the *right-hand* portion of the A scale, and the root contains *one-half* the number of digits in the original number.

Mention should be made of another method of extracting the square root, by which more accurate readings may generally be obtained. This involves the use of the C and D scales only. It is preferable to work in this case with the slide inverted, although it can be used with almost equal facility in the ordinary position. If there is an *odd* number of digits in the number, the *right* index, or if an *even* number of digits, the *left* index of the inverted scale C^1 is placed so as to coincide with the number on D of which the root is sought. Then by the aid of the cursor, the number is found on D which coincides with the same number on C^1, which number is the root sought.

Example 15. Find the square root of 222.

Setting the right-hand index of C^1 to 222 on D, it is found that 14·9 is the number on D which coincides with the same number on C^1. This therefore is the required square root (14·899 +).

Cubes and Cube Roots.

In raising a number to its third power, the foregoing method of squaring is used in conjunction with ordinary multiplication, the rule being expressed as follows:—

Rule for Cubing a Number. Bring the right or left-hand index of C to the given number on D, and over the same number on the left-hand B scale read the required cube on A.

Example 16. Find the cube of 1·25

Set the left-hand index of C to 1·25 on D, and over 1·25 on B read 1·95 on A [1·953125].

Fig 8 shows the position of the slide for this operation, and it will be noticed that in this case the slide projects to the right.

Example 17. Find the cube of 7·25.

In this case the right-hand index of C is set to 7·25 on D, when over 7·25 on B is read 381 on A. This setting of the rule is shown in Fig. 9, from which it will be seen that the slide in this case projects to the left.

By the above methods of working, four scales are brought into requisition. Of these the D scale and the left-hand scale of B are always employed, and are to be read as of equal denomination; while the final result is read off on the left or right-hand scales of A as the case may be. This method of cubing a number is generally the most satisfactory, although it is sometimes preferable to carry out the operation by continuous multiplication, using the cursor in the manner already explained.

In many cases the number of digits in the product is self-evident, but in any case the following rules are sufficient to determine the number of digits in any cube, n being the number of digits in the number, and N the required number in the cube:—

$N = 3n—2$ when the product is read on the left-hand scale of A with the slide projecting to the *right*.

$N = 3n—1$ when the product is read on the right-hand scale of A, slide projecting to the *right*.

$N = 3n$ when the product is read on the left-hand scale of A with the slide projecting to the *left*.

With decimals the same rules apply, but, as before, the number of digits must be read as —1, —2, etc., when one, two, etc., cyphers, follow immediately after the decimal point.

Thus in Example 16 the result is read on the *left*-hand scale of A, and with the slide projecting to the *right*; so that as there is one digit in the number there will be $3 — 2 = 1$ digit in the answer, which is therefore read as 1·95.

In Example 17 the result is read on the *left*-hand scale of A, with the slide projecting to the *left*. In this case, therefore, there

will be three digits in the answer, which is consequently read as 381 (381·078125).

CUBE ROOT.—The method of performing the inverse of the foregoing operation, which is necessary in order to obtain the cube root of a number, will now be evident. Employing the same scales as before, we have the following

RULE FOR EXTRACTING THE CUBE ROOT.—*Move the slide, either from right to left or from left to right, until under the given number on* A *is found a number on the left-hand* B *scale identical with the number which is simultaneously found on* D, *under the right or left-hand index of* C. This number is the required cube root.

But it will be found that by using the four scales mentioned above, it becomes possible, by working in the manner described, to find *three* values for the root. To decide which of these is the value sought, it is necessary (as in the arithmetical method of extraction) to point off the given number into sections of three figures, commencing at the decimal point and proceeding to the left for numbers greater than unity, and to the right for numbers less than unity. Then if the first section of figures on the left consists of 1 figure, the number is to be taken on the left-hand scale of A and the slide is to project to the *right*. If of 2 figures, the number is to be taken on the right-hand scale of A with the slide to the *right*. If of 3 figures, the number is to be taken on the left-hand scale of A, the slide projecting to the *left*.

Therefore in order to find the cube root of 4000, 4, 0·004, 0·000004, etc., 4 on the left-hand scale of A is used with the slide to the *right*-hand, since these examples evidently fall under the first rule.

For the cube root of 40,000, 40, 0·40, 0·00004, etc., the right-hand scale of A is used with the slide to the *right*.

And finally for 400,000, 400, 0·400, 0·000400, etc., the left-hand scale of A is used with the slide to the *left-hand*.

EXAMPLE 18. Find the cube root of 12.

This evidently falls under the second of the above conditions; so bringing the cursor to 12 on A, the slide is moved to the right until it is found that when the left-hand index (1) of C is over 2·29 on D, 2·29 on B is also under the line on the cursor.

The rule for the number of digits in the root will be obvious from these for cubing a number. Thus in the previous example, by adding 1 to the number of digits (2) and dividing by 3, it is seen that the required root must contain 1 digit. It is therefore read as 2·29 [2·289].

Another method of finding the cube root is indicated in Fig. 10. This involves the use of the inverted slide which, as shown, is placed so that what is now the right-hand index of the slide coincides with the number of which the cube root is sought, in this case 12. It is then necessary to find what number on D coincides with the same number on the inverted scale of B, and this in the case considered is seen to be 2·29, which is therefore the root sought.

Example 19. Find by this method the cube root of 130.

The disposition of the rule for the solution of this problem is shown in Fig. 11, from which it will be seen that to the number taken on the left-hand scale of A (since it consists of 3 digits) is placed the left-hand index of the slide, and the number on D, which coincides with a similar number on the inverted scale of B, is seen to be 5·07 [5·065] which is therefore the required root. It should be noted that when the cube root is found by means of the inverted slide, the latter projects in the opposite direction to that given in the first rule for extracting the cube root.

Fourth Powers and Roots.

Since to raise a number to its fourth power it is only necessary to square the square of a number, we have the following:—

Rule for Raising a Number to the Fourth Power. Set the right or left-hand index of C to the given number on D, and over the number on C read the fourth power on A.

The fourth root of a number may be extracted by reversing the operation, but it is generally preferable to extract the square root of the square root.

Practical Illustrations of the Use of the Calculating Rule.

In the preceding pages general instructions have been given for performing the operations most usually in request. A number of practical examples will now be given in order to exhibit the great utility of the Calculating Rule in a variety of applications.

Mensuration, &c.

EXAMPLE 20. Determine the area of a circle $5\frac{3}{4}$ inches in diameter.

In order to obtain the area of a circle of diameter d, we have the formula:— Area $= \frac{\pi}{4} d^2$.

The division of π by 4 is first effected by bringing 4 on B to the line π on A; then, without taking note of the result of this operation, the subsequent multiplication by d^2 is performed by reading the result on A over d on C.

Therefore in the example given, set 4 on B to π on A, as in Fig. 12, and over 5·75 on C read by the aid of the cursor 26 square inches on A, which is the area required. The true result is 25·967, thus showing that the result obtained is sufficiently accurate for all practical requirements.

EXAMPLE 21. Find the area in square *feet* of a circle 16 inches in diameter.

In this case the formula becomes

$$\text{Area} = \frac{\pi \times d^2}{4 \times 144} = \frac{\pi}{576} d^2$$

Therefore setting 576 on the left-hand scale of B to π on A, as in Fig. 13, we read over 16 on C the area $= 1·4$ [1·396] square feet on A.

Problems relating to the solidity or cubic contents of cylinders are readily solved by the aid of the Calculating Rule, special graduation lines being marked on the C scale which greatly facilitate calculations of this kind.

In order that these special lines may be used intelligently the principle upon which their positions are determined may be profitably explained.

The cubic contents of a cylinder = Sectional area × length

$$= \frac{\pi}{4} \times d^2 \times l$$

in which d is the diameter of the cylinder and l its length.

But the successive multiplication of $\frac{\pi}{4}$ by d^2 and then by l may be very conveniently avoided by substituting for $\frac{\pi}{4}$, its reciprocal, $\frac{4}{\pi}$ $= 1{\cdot}273$. The formula then becomes

$$\text{Cubic contents} = \frac{d^2 \times l}{1{\cdot}273}$$

in which form the solution of problems of this kind may be readily effected by one setting of the slide. Therefore on C a special line is marked at the position corresponding to 1·273 on B.

Example 22. Find the cubic contents of a cylinder 1·24 feet in diameter and 3·24 feet in length.

Place, as indicated in Fig. 14, the special graduation line c to 1·24 on D, and over 3·24 on B read 3·91 cubic feet on A as the required cubic contents.

It will be seen that if the setting of c to the given diameter necessitates the slide being drawn more than one-half of its length to the right, it is not possible to read off the contents in the manner described. Thus, if the given diameter was 6·15 inches and the length 36·5 inches, the setting of the slide as above directed would bring the slide too far to the right to enable the result to be read off. For this reason, therefore, a supplementary graduation marked c^1 is given on the C scale, the distance between the marks c and c^1 being equal to that from 1 to 10 on the upper scales. It will be understood that in using the graduation c^1 the result is to be multiplied by 10, since it is clear that the slide has been moved back to such an extent as to cause all the readings between the upper scales to be one-tenth of their correct value.

Example 23. Find the cubic contents of a cylinder 6·15 inches in diameter and 36·5 inches in length.

Set the special graduation line or "gauge point" c^1 to 6·15 on D, as shown in Fig. 15, and over 36·5 on B read 1084 cubic inches on A.

Example 24. Find the contents in cubic inches of a pipe 4·8 inches in diameter and 34 inches long.

Set, as in Fig. 16, the line *c* to 4·8 on D and over 3·4 (read 34) on B, read 615 cubic inches on A.

The determination of the number of digits in the answer in such problems is most expeditiously effected by a rough mental calculation. The methodical manner of deciding this point would be as follows, taking Example 24 as an illustration:—

Setting gauge point *c* on C to 4·8 on D we obtain $\frac{4{\cdot}8^2}{1{\cdot}273}$ on A over 1 on B. Reading over 3·4 on B, we should obtain 61·5 on A; but as the reading on B is, by the conditions of the problem, increased tenfold, we must likewise increase the reading of A tenfold, and therefore over 34 on B, read 615 on A.

Screw Cutting.

As already pointed out, the Calculating Rule is particularly serviceable in all calculations involving the proportion or ratio existing between various quantities. One of the most useful practical illustrations of this application of the instrument is found in solving problems relating to screw cutting and the determination of the change wheels necessary to cut a screw of given pitch.

EXAMPLE 25. In a lathe, the guide screw of which is of $\frac{1}{2}$ inch pitch, it is required to cut a thread of $\frac{3}{16}$ inch pitch. If a wheel containing 45 teeth is placed on the mandrel, what wheel must be used on the guide screw to cut the required thread, and, alternatively, what other pairs of wheels may be used if desired?

In this case if m be the number of teeth in the wheel on the mandrel, and s the number of teeth in the required guide screw wheel, the ratio $\frac{m}{s}$ must be such that

$$\frac{m}{s} \times \tfrac{1}{2} = \tfrac{3}{16} \text{ or } \frac{m}{s} = \tfrac{3}{8}$$

Therefore setting 8 on B to 3 on A, as in Fig. 17, the required ratio is so established that under any mandrel wheel on A is found the corresponding wheel for the leading screw on B.

Then as the wheel on the mandrel has 45 teeth, it is seen that the required wheel for the leading screw must contain 120 teeth.

Other pairs of wheels which will produce the required screw are seen to be $\frac{15}{40}$, $\frac{30}{80}$ and $\frac{60}{160}$ if the teeth in the change wheels are multiples of 5. Failing this restriction we have $\frac{21}{56}$, $\frac{27}{72}$, $\frac{36}{96}$ etc. as other sets of wheels which will cut the thread required.

Gearing, &c.

Example 26. Determine the circumference in *feet* of a pulley 27·5 inches in diameter.

In this case, the circumference of a circle 27·5 inches in diameter is to be divided by 12. Therefore set 12 on B to the special line marked π on A, as shown in Fig. 18, and over 27·5 on B read 7·2 feet on A.

Example 27. What is the speed of a belt which runs on a pulley 17 inches in diameter, when the latter makes 50 revolutions per minute?

Set 12 on B to π on A, and bring the cursor to 17 on B. Then bring the right-hand index of B to the cursor, and over 50 on B read 222·5 feet on A as the required belt speed.

Example 28. A pulley 5 feet in diameter, running at 225 revolutions per minute, drives by a belt another pulley 3 feet 7 inches in diameter. Find the revolutions per minute of the latter.

Problems of this kind are most readily dealt with by inverting the slide as shown in Fig. 19. Setting 60 on C^1 to 225 on D, by the aid of the cursor it will be found that under 43 on C^1 is 314 on D, this being the required number of revolutions of the small pulley.

Example 29. A spur wheel, 11 feet 5 inches in diameter at the pitch line, has 230 teeth. Find the pitch of the teeth.

Reducing the pitch diameter to inches (137), it is then only necessary to multiply by π (the ratio of the circumference of a circle to its diameter) and divide by 230.

Therefore, as indicated in Fig. 20, set 230 on B to 137 on A, and over the special graduation π on B read 1·87 inches on A as the required pitch.

Example 30. The velocity of a belt 8 inches in width is 4000 feet per minute. Find the horse power it will transmit, allowing an effective tension of 50 lbs. per inch of width.

In this case we have—

$$\text{Horse Power} = \frac{\text{Velocity} \times \text{tension} \times \text{width}}{33000}$$

which in the above example becomes—

$$\text{Horse Power} = \frac{4000 \times 50 \times 8}{33000} = \frac{4000 \times 8}{660}$$

Therefore set 660 on B to 4000 on A (Fig. 21), and over 8 on B read 48·5 on A, the required horse power.

It will be understood that with the slide so placed, the power transmitted by any width of belt at the given velocity and tension may be read off on A above the belt width on B.

EXAMPLE 31. Required the diameters of two spur wheels, one of which is to make 7 revolutions while the other makes 8, the distance apart of their centres being 5 feet.

Set 7 + 8 = 15 on C to 120 on D, as shown in Fig. 22, and under 7 and 8 on C read 56 and 64 inches as the respective diameters on D.

EXAMPLE 32. Find the horse power which can be transmitted by a spur wheel 5 feet in diameter and 3 inches pitch, when running at 70 revolutions per minute, assuming that

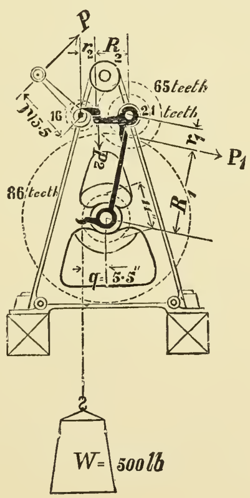
$$\text{Horse Power} = \frac{(\text{pitch})^2 \times \text{diam. in inches} \times \text{revolutions.}}{400}$$

which in the above example becomes—

$$\text{Horse Power} = \frac{3^2 \times 60 \times 70}{400}$$

Set 400 on B to 3 on D, as shown in Fig. 23, and bring the cursor to 60 on B. Then bring 1 on B to the cursor, and over 70 revolutions on B read 94·5 horse power on A, as shown in Fig. 24.

EXAMPLE 33. As an example of the more complicated problems which may be readily solved by the aid of the Calculating Rule, we take the hand winch shown in the accompanying cut, and proceed to



determine the power P which must be applied at the handle in order to raise a weight of 500 lbs., friction being neglected. The relative proportions of the gearing are shown in the sketch.

The hand lever p, 15·5 inches in length, turns a pinion of 16 teeth, which gears with a wheel of 65 teeth. On the same shaft as the latter is a pinion of 21 teeth gearing with a wheel of 86 teeth on the barrel shaft. The barrel is 11 inches in diameter. The thick lines in the diagram show very clearly the effective leverages which are acting.

Designating by P and W the power and weight respectively, by q the radius of the barrel, and by R_1 r_1 R_2 and r_2 the radii of the wheels, it will be seen that

(1) The pressure on the teeth at $P_1 = W \times \frac{q}{R_1}$

(2) „ „ „ $P_2 = P_1 \times \frac{r_1}{R_2}$

(3) The power applied at the handle $= P = P_2 \times \frac{r_2}{p}$

From this it follows that

$$P = W \times \frac{q}{p} \times \frac{r_1}{R_1} \times \frac{r_2}{R_2}$$

and as we may substitute the number of the teeth in the wheels for their radii, we have

$$P = 500 \times \frac{5{\cdot}5}{15{\cdot}5} \times \frac{21}{86} \times \frac{16}{65}$$

The manner in which this calculation is effected is shown by the successive positions of the slide and cursor in Figs. 25 to 28.

As indicated in Fig. 25, the slide is first set so that 1 on B coincides with 5 (read 500) on A, and the cursor set so that the line thereon agrees with 5·5 on B. The product of $500 \times 5{\cdot}5$ could now be read off on A under the cursor line, but this intermediate result is not read but is at once divided by 15·5, by bringing this latter on B to agree with the cursor as shown in Fig. 26.

The result of this operation $\frac{500 \times 5{\cdot}5}{15{\cdot}5}$ could now be read off on A over 1 on B, but instead, it is multiplied by 21 by bringing the cursor to the position shown. Bringing 86 on the slide to the cursor, as shown in Fig. 27, effects the succeeding division, and placing the cursor to agree with 16 on B, and moving the slide so that 65 on B coincides with the cursor, as in Fig. 28, the result may be read off on A over 1 on B.

From this it is seen that

$$P = \frac{500 \times 5{\cdot}5 \times 21 \times 16}{15{\cdot}5 \times 86 \times 65} = 10{\cdot}66 \text{ lbs.}$$

EXAMPLE 34. Find the horse power transmitted by a steel shaft 5 inches in diameter at 75 revolutions per minute, from

$$\text{H.P.} = \frac{d^3 \times \text{N}}{60}$$

in which d is the diameter in inches, and N the number of revolutions per minute.

Set 10 on C to the diameter (5) on D as shown in Fig. 29, and bring cursor to 5 on B. Next bring the divisor 60 to the cursor, and over 75 on B read 156 H.P. on A (Fig 30). The true result is 156·25.

Delivery from Pumps, &c.

EXAMPLE 35. Find the amount of water in gallons delivered per stroke (theoretically), by a pump 6 inches in diameter and 1·2 feet stroke.

Set 29·4 on B to 6 on D by the aid of the cursor (Fig. 31), and over 1·2 on B read 1·47 gallons per stroke on A.

$$\left(\text{N.B.} \quad \frac{1}{29{\cdot}4} = \frac{\pi}{4} \times \frac{12}{277}\right)$$

In practice a suitable allowance must be made for slip and other losses of efficiency.

EXAMPLE 36. Find the theoretical velocity of water flowing under a head of 55 feet.

Set 100 on B to 55 on A as in Fig. 32, and under 64·4 on B read, by the cursor, 59·5 feet per second on D as the required velocity.

In this and similar cases in which the right-hand index of B must be used in place of the left-hand index, it must be understood that results on A being diminished one hundredfold, readings on D are to be increased tenfold. Thus in the last example the reading on D is 5·95, but as the conditions of the problem render necessary the use of the right-hand index (100) of B, the result is increased tenfold and is read as 59·5.

Power of Steam Engines.

EXAMPLE 37. Find the indicated horse power of an engine in which the cylinder diameter (d) = 18 inches ; the stroke (s) = 2 feet ; the mean effective pressure in lbs. per square inch (p) = 40 ; and the number of revolutions per minute (n) = 60.

The usual formula for this calculation is

$$\text{I.H.P.} = \frac{d^2 \times {\cdot}7854 \times s \times 2 \times n \times p}{33000}$$

an expression which may be conveniently changed to

$$\text{I.H.P.} = \frac{d^2 \times s \times n \times p}{21000}$$

To find the indicated horse power in the above example, 21000 on B is set to the cylinder diameter (18) on D as shown in Fig. 33, and the cursor brought to agree with the stroke in feet (2) on B. Then 1 on B is brought to the cursor (Fig. 34) and the latter to the revolutions per minute (60) on B. 1 on B is then brought to the cursor, and over the mean effective pressure (40) on B is read the required I.H.P. = 74 on A (Fig. 35).

Example 38. Find the tractive force of a locomotive having driving wheels 54 inches in diameter, and cylinders 18 inches in diameter, with a stroke of 24 inches.

Set 54 on B to 18 on D as in Fig. 36, and over 24 on B read 144 on A as the tractive force in lbs. for each lb. of mean effective pressure on the piston.

The Trigonometrical Applications of the Calculating Rule.

The *Scale* of *Sines*.

On the under side of the slide of the Calculating Rule will be found three scales. The upper of these, marked S, is the Scale of Sines, and is used to determine the natural sines of angles of from 35 minutes to 90 degrees. The notation of this scale will be evident on inspection. The main divisions 1, 2, 3, etc., represent the degrees of angles; but the values of the sub-divisions differ according to their position on the scale. Thus if any primary space is sub-divided into 12 parts, each of the latter will be read as five minutes (5′).

In order to determine the sine of an angle by the aid of the Calculating Rule, the slide is withdrawn from the stock of the instrument and re-inserted in such a manner that the scale marked S lies adjacent to the A scale of the rule, and with the right and left-hand indices coinciding.

With the slide so placed, the sine of any angle may be read off on the A scale, immediately above the angle on the scale S. Fig. 37 gives a number of examples which will serve to illustrate the method of procedure, the following readings being marked thereon:—

Sine 0°40′	=	0·0116	Between 1 and 10 of the A scale.	Sine 6°	=	0·1045	Between 10 and 100 of the A scale.
Sine 1°	=	0·0174		Sine 7°30′	=	0·1305	
Sine 1°50′	=	0·0319		Sine 9°50′	=	0·1707	
Sine 2°10′	=	0·0378		Sine 12°20′	=	0·214	
Sine 3°	=	0·0523		Sine 15°	=	0·259	
Sine 3°20′	=	0·0581		Sine 21°30′	=	0·367	
Sine 4°20′	=	0·0756		Sine 30°	=	0·500	
Sine 5°	=	0·0872		Sine 56°	=	0·829	
Sine 5°40′	=	0·0987		Sine 74°	=	0.961	

It must be carefully noted that the value of the sines of all angles read on the first half of the A scale, *i.e.*, between 1 and 10, are such that the decimal point is always followed by 0; while those read on

the right-hand half of the A scale, between 10 and 100, are prefixed by the decimal point only. This will be evident from the example shown in Fig. 37.

The method of calculating with the sine scale is precisely similar to that already explained in the preceding pages. A few illustrations will, however, serve to exemplify the many varied uses which can be made of the instrument for trigonometrical calculations.

Example 39. Find the product of sine $5°50' \times 38$.

Placing 1 of the sine scale under 38 on A, as shown in Fig. 38, the product is found above the line representing the angle 5°50′ on the sine scale. The required product is then read off as 3·86.

Example 40. Find the product of $7{\cdot}4 \times$ sine 15° (see Fig. 39).

In this case the result cannot be directly read off, as it falls beyond the rule. It is therefore necessary to proceed as previously explained, by placing the right-hand index of S to 7·4 on A, when over 15° on S is read 1·917 on A.

Example 41. Find the result of dividing 20 by sine 5°40′.

As shown in Fig. 40, 5°40′ on the S scale is placed to 20 on scale A, and over the left-hand index of S is read 202·6, the required quotient.

Example 42. Given, two angles of a triangle,

$$\alpha = 20°30'$$
$$\beta = 45° \quad ;$$

also, the length of the side opposite to the angle β

$$b = 75$$

find the length of the side a, opposite to the angle α

The formula for this is:—

$$\alpha = \frac{75 \text{ sine } 20°30'}{\text{sine } 45°}$$

Therefore in order to solve this problem, the line representing 45° on scale S is placed so as to agree with 75 on A, and the result, $37{\cdot}15 =$ length of side a will be found on A, over 20°30′ on S (Fig. 41).

Another method of determining the sines of angles may be employed, in which the reversal of the slide is avoided. It will be found that the back of the rule has recesses or apertures at the two extremities, and that upon the edges of these are index lines. By the aid of the upper of these the sines of angles may be found in the following manner:—

Draw the slide to the right (Fig. 42), until the angle, on the sine scale S, coincides with the index line in the aperture. If the rule is

now turned face upwards, the required sine will be found on B, under 100 of A. For example, sine $2°30' = 0·0437$. Also:—

Sine $79° = 0·982$
Sine $60° = 0·866$
Sine $28° = 0·469$
Sine $13° = 0·2249$
Sine $6° = 0·1045$
Sine $2° = 0·0349$

With this method of determining the sines of angles, the various calculations may be effected quite as readily as with the slide reversed.

EXAMPLE 43. Find sine^2 20°.

In this case (Fig. 43) the sine of 20° is found as before on the A scale opposite 20° on the S scale of the reversed slide, the value being read 0·342. The cursor is then brought to this value on the D scale and the value of sine^2 20° is read off on A at the cursor line. This is found to be 0·1169.

When the sine is known and the corresponding angle is to be ascertained, the mode of operating will be evident from the foregoing, and further explanation is therefore unnecessary.

The Scale of Tangents.

The lowest of the three scales on the reverse of the slide is marked T, and is used for determining the tangents of given angles.

One method employed is very similar to that used for finding the sines, the slide being reversed and re-inserted as before. The respective tangents of the angles marked on scale T may then be read off on scale D.

It should be noted, however, that with the 10 in. rule only those tangents which are between 0·1 and 1 can be directly read off, these limiting values corresponding to angles of 5°43′ and 45°.

In Fig. 44 the following values are shown:—

tangent 6°20′	= 0·1109	tangent 25°	= 0·4663
tangent 7°	= 0·1228	tangent 27°40′	= 0·5243
tangent 9°50′	= 0·1733	tangent 30°	= 0·5774
tangent 13°	= 0·2309	tangent 37°30′	= 0·7673
tangent 18°30′	= 0·3346	tangent 40°10′	= 0·8441
tangent 20°10′	= 0·3673	tangent 44°30′	= 0·9827

If the angle whose tangent is required is greater than 45°, the method of operation is modified in the manner illustrated by the example given in Figs. 45 and 46.

EXAMPLE 44. Find tan. 60°.

For this we have:

$$\tan.\ 60° = \frac{1}{\tan.\ (90°—60°)} = \frac{1}{\tan.\ 30°} = \frac{1}{0·5774} = 1·732$$

EXAMPLE 45. Find 240 × tan. 60°.

$$\text{Answer: } 240 \times \tan.\ 60° = \frac{240}{\tan.\ 30°} = \frac{240}{0·5774} = 415·5$$

The various calculations in which the tangents of angles are involved are carried out in the same way as described in connection with the calculations involving the sines of angles.

The tangents of angles may also be obtained with the slide in its usual position. This is generally more convenient than the method described above, since further multiplication or other treatment of the tangent may be carried out directly. Moreover the tangents of angles greater than 45° may be found more conveniently.

With this method, the angle on Scale T is set to the index mark in the aperture at the left-hand end of the rule, the value of the tangent being read on C over 1 on D.

If the angle whose tangent is required is greater than 45°, we proceed as follows:—

Find tan. 60°:—Set 90°—60° = 30° on T to the index, and under 10 on C read 1·732 on D.

The tangents of angles less than 5°43′ cannot be read directly from the 10 in. rule, but the corresponding sines may be substituted for them without material error, as the discrepancy only affects the fourth decimal place. Thus

tan. 1°10′ = sin. 1°10′ = 0·020361

tan. 3°40′ = 0·06408. Sin. 3°40′ = 0·063952

When large numbers have to be multiplied, the discrepancy arising from this method becomes more serious, but in the case of angles between 3°30′ and 5°43′ it may be corrected by adding to the sine of the angle, 3 per cent. of the value of the sine. By this correction a sufficiently exact value of the tangent is obtained. With the 21 in. rule, the correct values may be directly obtained down to an angle of 0°35′.

The Scale of Logarithms.

The evenly-divided scale found in the centre of the reverse side of the slide and marked L, enables the logarithms of numbers to be

found. It must be observed, however, that only the *mantissa* of the logarithm can be read off, the characteristic being determined by the following rule:—

The characteristic of a logarithm is equal to the number of digits in the number, *minus* 1. If the number is wholly decimal, the index is equal to the number of ciphers following the decimal point *plus* 1. In the latter case the characteristic is *negative*, and is so distinguished by having the *minus* sign written over it.

The method of determining the logarithm of a given number is as follows:—

Referring to Fig. 47, the left-hand index of the C scale is set to the given number on the D scale. The rule is then turned over and the mantissa of the logarithm read on L at the index line in the aperture at the right-hand end of the rule. Thus—

$$\log.\ 358 = 2{\cdot}554 \qquad \log.\ 35{\cdot}8 = 1{\cdot}554$$
$$\log.\ 0{\cdot}358 = \bar{1}{\cdot}554 \qquad \log.\ 0{\cdot}0358 = \bar{2}{\cdot}554$$

The number corresponding to a given logarithm may be readily determined by a reversal of the above method.

Higher Powers and Roots.—One of the most useful applications of this method of obtaining the logarithms of numbers is found in determining powers and roots other than squares and cubes. Any power or root may in this way be very readily determined, the method to be followed being illustrated by the following example:—

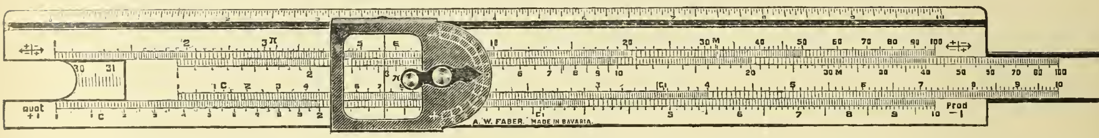
Example 46. Find $\sqrt[7]{1893}$. (See Figs. 48 and 49.)

Proceeding as above directed, it is found that log. $1893 = 3{\cdot}277$, and by the rule for division it is found that $\frac{3{\cdot}277}{7} = 0{\cdot}468$. Setting 468 on the scale of logarithms to the index line in the aperture, the number corresponding to log. $0{\cdot}468$ is found to be $2{\cdot}94$, this being found on the scale D under the left-hand index of the C scale. Therefore $\sqrt[7]{1893} = 2{\cdot}94$.

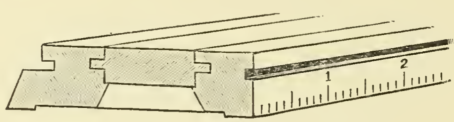
A. W. FABER'S

IMPROVED CALCULATING RULE.

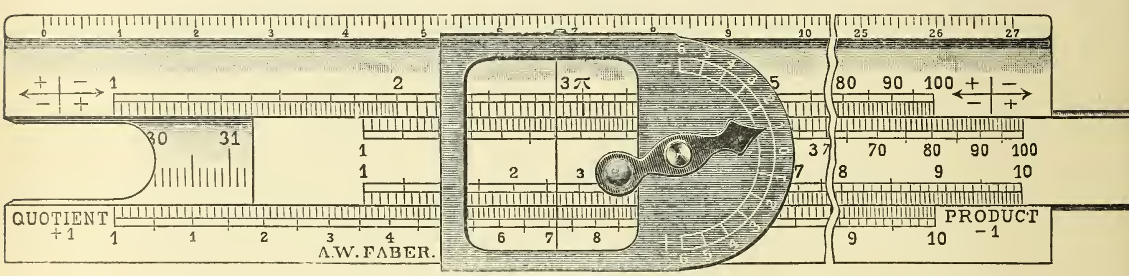
11 in. LONG, WITH DIGIT REGISTERING CURSOR.



REDUCED SIZE.



SECTIONAL VIEW, ACTUAL SIZE.



PORTION OF RULE, ACTUAL SIZE.

One of the chief difficulties met with in using the Calculating Rule is the determination of the number of digits in the final results of the several calculating operations. There are several methods by which the position of the decimal point can be determined, but generally speaking these are complicated and difficult to remember. In the improved form of A. W. FABER'S Calculating Rule a set of abbreviated instructions is provided in the form of four inscriptions—two at each end of the Rule—and by the aid of these very simple mnemonics, the remembrance of the somewhat complicated rules is so greatly facilitated that the possibility of any error arising is exceedingly remote.

This form of Slide Rule is made longer at each end by about $\frac{3}{8}$ in., this extra length affording a firmer hold for the cursor when used near the extreme ends of the scales. On the spaces on the rule face thus provided are placed the four inscriptions above mentioned. Of these signs, two are placed on the lower scale (D) of the rule, and two on the upper scale (A) of the rule. Of the former, the expression $\frac{\text{Quotient}}{+1}$ is found at the left-hand end of the lower scale, while the expression $\frac{\text{Product}}{-1}$ is found at the right hand extremity of the same scale. On the upper or A scale, the inscriptions at each end are alike, being of the form $\xleftarrow[-]{+}\Big|\xrightarrow[+]{-}$ The significance of these inscriptions will be shown in the appended examples.

An addition has also been made to the sliding cursor, which is now provided with a semi-circular graduated scale; this, as will be seen, is divided into 12 equal parts. The graduations which commence at the centre mark 0, proceed upwards to —6 (minus values) and downwards to + 6 (plus values). A small pointer is provided, which can be set to any of the graduations of the cursor scale.

The rules for the inscriptions, $\frac{\text{Product}}{-1}$ and $\frac{\text{Quotient}}{+1}$ are only to be used in conjunction with the lower scale; the signs $\xleftarrow[-]{+}\Big|\xrightarrow[+]{-}$ can be used for both the upper and lower scales.

LOWER SCALE.

Rule for Number of Digits in a Product.

The number of Digits in a product is equal to the sum of the digits in the two factors if the result is obtained on the left of the first factor; if, however, the result is found to the RIGHT of the first factor, the number of digits in the result is equal to the sum of the digits in the two factors MINUS 1.

The sign $\frac{\text{Product}}{-1}$ provides a visible memorandum of the above instruction.

In applying this rule it must be observed that in decimal values, when the first significant figure does not immediately follow the decimal point, the minus sign is to be prefixed to the number of digits, as many digits being counted *minus* as there are 0's following the decimal point.

Thus 35 has 2 digits; 3·5 has 1 digit; 0·35 has 0 digits; 0·035 has —1 digit; 0·0035 has —2 digits and so on.

EXAMPLE 47. $18 \times 3{\cdot}5 = 63.$

Sum of digits in factors... $= 2 + 1 = +3$
Product being on the right, deduct $1 = —1$

Number of digits in result ... $= +2$

EXAMPLE 48. $60 \times 35 = 2100.$

Sum of digits in factors ... ... $2 + 2 = +4$
Products being on left, no deduction $= 0$

Number of digits in result ... ... $= +4$

EXAMPLE 49. $0{\cdot}0014 \times 1{\cdot}7 = 0{\cdot}00238.$

Sum of digits in factors $= —2 + 1 = —1$
Product being on the right, deduct $1 = —1$

Number of digits in result ... ... $= —2$

When more than two factors are involved, the same method is followed, the several factors being dealt with in succession. It is here that the digit-registering cursor finds its application, the pointer being moved one division (commencing at 0) on the negative side of the scale each time an intermediate product is read on the right. The resulting number on the cursor-scale is to be subtracted from the sum of all the digits in all the factors employed.

Rule for the Number of Digits in a Quotient.

If the quotient is obtained with the result appearing on the right of the dividend, the number of digits in the result is found by subtracting the digits in the divisor from those in the dividend; but if the quotient is read on the LEFT, one is to be ADDED to the number of digits obtained as above.

The sign $\underset{+\,1}{\text{Quotient}}$ provides a visible memorandum of the above instruction.

EXAMPLE 50. $1440 \div 32 = 45.$

Difference in number of digits $4 — 2 = +2$
Quotient being on the right, no addition $= 0$

Number of digits in result ... ... $= +2$

EXAMPLE 51. $600 \div 32 = 18{\cdot}75$.

Difference in number of digits	$= 3 - 2 =$	$+1$
Quotient being on left, add 1 ...	$=$	$+1$
Number of digits in result	$=$	$+2$

EXAMPLE 52. $0{\cdot}0765 \div 51 = 0{\cdot}0015$.

Difference in number of digits	$= -1 - 2 =$	-3
Quotient being on left, add 1 ...	$=$	$+1$
Number of digits in result	$=$	-2

Combined Multiplication and Division.

In the foregoing examples of simple multiplication and division, mistakes are far less likely to be made than in complicated calculations involving both rules. With the addition of the new digit-registering cursor much greater facility is afforded for obtaining the results of such calculations quickly and accurately. The following example will serve to illustrate the method of procedure.

EXAMPLE 53. $$\frac{12 \times 8{\cdot}4 \times 21 \times 275}{48 \times 1{\cdot}5 \times 7{\cdot}7} = 1050$$

Note.—No notice is taken of decimals in working out this example, nor of the number of digits in the intermediate results, which latter are given below for the purpose of illustration only and ordinarily would not be noted.

(1) $12 \div 48 = 25$. Quotient is on right; therefore pointer remains untouched at 0.

(2) $25 \times 84 = 21$. Product is on left; pointer remains at 0.

(3) $21 \div 15 = 14$. Quotient is on left; therefore $+1$ is noted by moving pointer to $+1$ on scale.

(4) $14 \times 21 = 294$. Product is on right; therefore -1 is noted by moving pointer back to 0.

(5) $294 \div 77 = 382$. Quotient is on right; hence pointer is not moved.

(6) $382 \times 275 = 105$. Product is on left, and hence pointer is not moved.

At the end of the calculation, the pointer is on 0. Hence the number of digits in the result is equal to the sum of the digits in the denominator factors subtracted from the sum of the digits in the numerator factors.

Number of digits in denominator ...	=	+8
Number of digits in numerator (deduct)	=	+4
Number of digits in result	=	+4

Therefore the result must be read 1050.

The cursor scale is divided into 12 divisions only, in order to avoid the graduations being too small. This range will however be found sufficient for all ordinary calculations.

UPPER SCALE.

Explanation of the Sign $\leftarrow\pm|\mp\rightarrow$

With the divisions and numbering of the upper scales of the Calculating Rule, only calculations with numbers between 1 and 100 can be carried out, if we are restricted to the figured values of graduations of these scales. But as most cases would fall outside these limits, the use of the instrument would be very circumscribed were it not that by a simple method these limits can be indefinitely extended in either direction. For instance, if the number 4550 is to be dealt with, we see that this actual number is not to be found on the rule; but there is 45·50, that is to say, the number which is required, but with the decimal point moved two places to the left. If a multiplication is carried out with the latter number the result will be too small by two places of digits. Therefore at the end of the operation, the decimal point requires to be moved two places to the right, *i.e.*, to the number of digits read off on the rule, two have to be added in order to obtain the correct result. Again, if the number dealt with is 0·455, the point will have to be moved two places to the right in order to read the value on the rule, so that in the result, the point will be moved two places to the left, *i.e.*, from the number of digits read off at the end of the calculation, two places have to be subtracted in order to obtain the correct result. The method of procedure for division is the same as for multiplication, excepting that shifting the decimal point of the divisor to the left increases the result, and consequently, from the number of digits read off at the end of the calculation, as many are to be subtracted as the places by which the point was shifted to the left. Exactly the reverse procedure is followed if at the beginning the decimal point was moved to the right.

The sign $\overset{+}{\underset{-}{\leftarrow}}\big|\overset{-}{\underset{+}{\rightarrow}}$ represents a fraction, the vertical line indicating the positon of the decimal point, and the two arrows showing the direction in which the point must be regarded as having been shifted in order to read the values on the slide rule. It will now be seen that this sign completely indicates the method explained above, and which must be applied in order to obtain the correct number of digits in any calculation.

If in a multiplication, the point moves to the *left*, then to the number of digits read off as many must be *added* as the number of places by which the point was moved.

If the point moves to the *right*, these places must be *subtracted.*

If in a division, the point in the divisor is moved to the *left*, then these places must be *subtracted* at the end of the operation.

If the point moves to the *right*, then these places must be *added* at the end of the operation.

If now the shifting of the point to the negative or positive side is marked by means of the pointer and cursor scale before described, the final position of the pointer will exactly indicate the number of places by which the result, as read, must be increased or diminished in order to obtain the actual result.

Multiplication.

Example 54. 255 × 156 = 39780.

Using 2·55 on the upper scale of the rule, the decimal point is moved 2 places to the left; therefore set pointer to + 2. Multiply by 15·6; the point is moved by 1 place to the left, giving + 1, and the pointer at + 3. We read off 39·78, but as the pointer is at + 3, the result is 39780.

Eexample 55. 4680 × 0·0055 = 25·74.

Use 4·68 on the upper scale; point moved 3 places to the left, hence pointer to + 3. Multiply by 5·5; point moved 3 places to the right. We have, therefore, —3, and the pointer returns to 0. We read off 25·74, and as the pointer stands at 0, this reading is correct.

Division.

Example 56. 0·986 ÷ 425 = 0·00232.

Using 9·86 on the upper scale, the point is moved 1 place to the right; hence, as 0·986 is the dividend, we set the pointer to —1.

Dividing by 4·25, the point is moved 2 places to the left; hence the pointer is moved by — 2, since 425 is the divisor of the fraction. The result read off is 2·32, and as the pointer stands at — 3, the result will be 0·00232.

EXAMPLE 57. $3425 \div 0{\cdot}75 = 4566{\cdot}6$.

Using 34·25 on the upper scale, the point is moved 2 places to the left; hence pointer to + 2. Dividing by 7·5 the point is moved 1 place to the right, giving + 1. The pointer is therefore at + 3, and as we read off 4·5666, the true result will be 4566·6.

Here it may be pointed out that in multiplication we commence at 1 on the rule; in division the result is to be read at 1. In complicated calculations, it may occur that the multiplication is made with 100 as the initial position; but in that case the result must naturally be multiplied by 100 as often as the multiplication is made from 100. In the same way in division, if the reading is taken at 100, the result must be divided by 100 as often as the reading is taken at that graduation. It will be evident that this can be marked at once by an initial setting of the pointer, as will be shown in the following examples. We repeat examples 54 and 56 in order to show the modified procedure.

EXAMPLE 58. $255 \times 156 = 39780$.

Set 100 on the slide to 25·5 on the rule, and as the point is moved 1 place to the left, the pointer is set to + 1. As the setting is to 100, the result must be muliplied by 100 or the point moved by 2 places to the right, giving + 2, and consequently the pointer at + 3. Above 15·6 on the slide (point 1 place to left giving + 1 and pointer to + 4) we read 3·978 on the rule, and as the pointer is at + 4, the result is read as 39780.

EXAMPLE 59. $0{\cdot}986 \div 425 = 0{\cdot}00232$.

Under 9·86 on the rule we set 42·5 on the slide; the point of the dividend is moved 1 place to the right; hence pointer to —1. The point of the divisor is moved 1 place to the left, giving — 1 and the pointer at —2. Over 100 on the slide we read 23·2 on the rule; hence the result = 0·232. But as the reading was taken at 100, the result must be divided by 100, or the point moved two places to the left. The true result will therefore be 0·00232.

In order to avoid any error, it is preferable, before reading the result over 100, to shift the pointer at once by — 2, so that in the previous example, the pointer would be found finally at — 4.

Mixed Calculations.

In proportion and calculations involving combined multiplication and division, we may proceed by first registerng the movements of the decimal points, by means of the pointer; then the calculation can be carried out directly with the slide, only the reading off and the settings at 100 requiring to be taken into account and registered.

Squares and Square Roots.

The method may also be used for ascertaining the number of digits in squares. It is to be observed, however, that the pointer is to be set to the double value of the number of places through which the decimal point was moved for the purpose of squaring.

In calculating square roots, the procedure is exactly the same as in the ordinary extraction of roots. The number under the sign is marked off into sections of two figures each, starting from the decimal point and moving to the right or left, each section corresponding to 1 unit on the pointer scale.

For squaring and extracting square roots, the marking of the upper scale, 1 to 100, is very convenient, as over any number on the lower scale is its square on the upper scale.

Cubes and Cube Roots.

In cubing, the pointer must be set to triple the number of places which the decimal point requires to be moved in order that the number to be cubed can be read on the lower scale. Further, if 10 on C must be used to make the setting, the pointer must be moved by + 2 in addition, as explained for multiplication.

With the aid of the pointer, the number of places in cube roots can likewise be determined. As this calculation is possible without reversing the slide, the procedure will be further explained by the aid of examples. It will be noted that the cube roots of numbers between 1 and 100 are to be read directly on the lower scale, under 1 on the slide; while the cube roots of numbers between 100 and 1000 are to be found on the lower scale under 10. All other cases can be reduced to one or other of these cases by marking off the numbers into sections of three figures (starting from the decimal point and moving to the left and right), each section corresponding to 1 place on the pointer scale.

Example 60. $\sqrt[3]{479} = 7{\cdot}82.$

Set the cursor to 479 on the upper scale of the rule between 1 and 10, and move the slide to the left until, on the upper scale of the slide under the cursor line, and on the lower scale of the rule under 10, the same number (7·82) appears.

Example 61 $\sqrt[3]{89} = 4{\cdot}46.$

Set the cursor to 89 on the upper scale of the rule and move the slide to the right until the same number (4·46) appears on the upper scale of the slide under the cursor line, and on the lower scale of the rule under 1 on the slide.

Example 62. $\sqrt[3]{657{,}500} = 86{\cdot}9.$

Mark off from right to left sections of 3 figures as shown, and set the pointer to + 1. Then the cube root of 657·5 is to be found. Set the cursor line to 657·5 on the upper scale of the rule, between 1 and 10, and move the slide to the left until the same number (8·69) appears under the cursor line on the upper scale of the slide and under 10 on the lower scale of the rule. As the pointer is at + 1, the result is 86·9.

Example 63. $\sqrt[3]{0{\cdot}004{,}75} = 0{\cdot}1681.$

Mark off sections of three figures from the decimal point to the right, and set the pointer to — 1; then the cube root of 4·75 is to be found. Set the cursor to 4·75 on the upper scale of the rule, and move the slide to the right, until the same number (1·681) appears on the upper scale of the slide under the cursor line, and on the lower scale of the rule under 1 on the slide. As the pointer is at — 1, the result is 0·1681.

No further explanation is necessary to show that with the aid of this simple contrivance, which entirely relieves the memory, very complicated mixed calculations can be carried out with certainty, and the number of digits in the result determined without difficulty.

Circumference and Area of a Cylinder.

On the upper scale of the slide of this Slide Rule will be found a gauge-point, marked M, placed at the graduation corresponding to $\frac{100}{\pi} = 31{\cdot}83.$ By the aid of this gauge-point it becomes possible to read off the circumference and curved area of a cylinder with a single movement of the slide. The point M on the B scale is set to the diameter of the cylinder on the A scale, and the circumference is

read over 1 or 100 of B, depending upon whether the diameter is greater or less than 31·83. The curved area of the same cylinder is found on the A scale, over the length on scale B.

Example 64. Find the circumference and curved area of a cylinder 2·7 ft. in diameter and 6·5 ft. in length.

Placing the gauge-point M to 27 on A (see Fig. 50, Plate 13), the circumference ($d \times \pi$) is found over 100 to be 8·47 ft.; while over the length 65 on B is read the curved area ($d \times \pi \times l$) = 55·1 square ft. on A. The position of the decimal point is evident, since it is obvious that a cylinder of the dimensions given could not have a circumference of 84·7 ft. but only 8·47. Similarly the area can only be 55·1 square ft., and not 5·51.

Note.—The gauge-point M was placed at 31·83 instead of at 3·183, in order to avoid confusion with the gauge-point π (3·14159).

Sines and Tangents.

This Rule is provided with an aperture at the left-hand extremity, enabling Sines and Tangents of angles to be read off directly without reversing the Slide.

Example 65.—Sine 5°20′ = 0·0929. Turning to the back of the Rule the slide is withdrawn to the left as shown in Fig. 51, Plate 13, until the angle 5°20′ on the scale S agrees with the index mark in the aperture. Turning the rule over, the required Sine is found under the left index of the A scale, to be 0·0929.

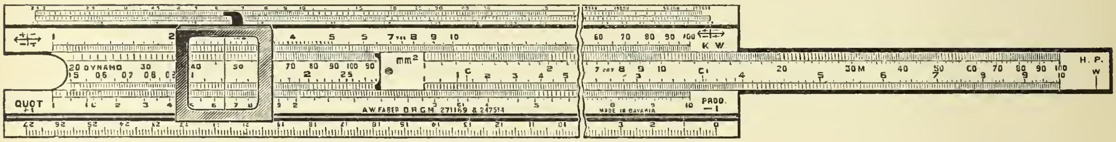
Example 66.—Tangent 12°40′ = 0·2247. Turning to the back of the rule as before, the slide is withdrawn to the left, as shown in Fig. 52, Plate 13, until the angle 12°40′ on the scale T agrees with the lower index in the aperture. Turning the rule over, the required Tangent is found, over the left index of the D scale, to be 0·2247.

Mistakes in reading these scales will be avoided if it is borne in mind that all the readings are found on the *slide*, and that on the back of the slide the Sines are "above" and the Tangents "below," while the corresponding readings on the face of the slide are also Sines "above" and Tangents "below."

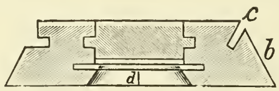
A. W. FABER'S Improved Calculating Rule for Electrical and Mechanical Engineers,

With LOG=LOG *Scale.*

11 IN. LONG WITH SPRING BACK AND SPECIAL CURSOR.



368. REDUCED SIZE.



SECTIONAL VIEW, ACTUAL SIZE.

This New Calculating Rule is exclusively intended for Mechanical and Electrical Engineers, but all those calculations which can possibly be made with the standard Calculating Rule, can also be carried out with it. Only small modifications have been made in the construction of the rule, but it nevertheless results that three other important calculations can be made with it. In the new arrangement both sides are bevelled, while the left end of the slide is fitted with a metal covering piece or index, having a sharp edge; this is used in conjunction with new scales on the bottom of the groove of the rule *d*, which take the place of the measuring scale in the ordinary form of rule. As will be seen from the illustration, the New Calculating Rule is of almost exactly the same form as the previous construction.

Log=log *Scales.*

The bevelled side, *b*, carries two sets of graduations, side by side—the so-called "log-log" scales. These two sets of graduations form a continuous log-log scale running from 1·1 to 100,000. The first half runs from 1·1 to 2·9, and the second half from 2·9 to 100,000. The cursor carries a projecting metal tongue, the end of which serves as a marker and corresponds exactly with the line on the cursor glass. With this arrangement, the log-log scale can be used in conjunction with the

lower scale on the slide, and powers and roots within the limits of 1·1 and 100,000 and of the form a^x or $\sqrt[x]{a}$ can be obtained, in which neither a nor x need be a whole number. The choice of the lower limit, 1·1 was determined by considerations of the most usually occurring values, and the length of the scale then determined the upper limit, 100,000. Both parts of the log-log scales extend somewhat beyond the limits of the lower scale of the rule; while on the lower scale of the slide, to the right of 10, is a special index mark W, the length 1—W being equal to that of the lower log-log scale.

Involution (Raising to Powers).

EXAMPLE 67. $1{\cdot}124^{2{\cdot}24} = 1{\cdot}299$.

Set the metal tongue of the cursor to the root 1·124, on the log-log scale, and bring 1 on the lower scale on the slide under the line on the glass; then set the line on the glass to the exponent 2·24 on the lower scale of the slide, when the power, 1·299 can be read off on the log-log scale, under the metal tongue of the cursor.

As will be seen from the above example, the reading of results which do not exceed 2·9 is very simple. But if the number, when raised to the given power, exceeds 2·9, then the special index mark W is used in place of 1 and the result is found on the second or upper half of the log-log scale.

EXAMPLE 68. $1{\cdot}665^{3{\cdot}17} = 5{\cdot}03$.

As before, set the cursor to 1·665 on the log-log scale, bring W on the lower scale of the slide under the cursor line, set the cursor to the exponent 3·17 on the lower scale of the slide and read the power, 5·03, under the tongue of the cursor, on the second log-log scale.

Evolution (Extracting Roots).

EXAMPLE 69. $\sqrt[2{\cdot}8]{26{\cdot}5} = 3{\cdot}22$.

Set the cursor to 26·5 on the log-log scale, bring the exponent 2·8 on the lower scale on the slide under the cursor line, and after setting the cursor to 1 on the lower scale on the slide, read off the root, 3·22, on the upper log-log scale.

If the root is less than 2·9, the special index mark W is used, and the result is found in the first log-log scale.

Example 70. $\sqrt[7\cdot15]{8\cdot75} = 1{,}354$.

Set the cursor to 8·75 on the log-log scale; bring the exponent 7·15 on the lower scale on the slide under the cursor line, the cursor to the index mark W, and read off the root, 1·354 on the lower log-log scale.

If in extracting roots the result falls below 1·1, the calculation cannot be made with this slide rule; neither can numbers less than 1·1 be raised to powers.

Graduations on the Bottom of the Calculating Rule.

The measuring scale omitted from the bottom side *d* of the Calculating Rule has been replaced by two new sets of logarithmic graduations, which are read by means of the metal indicator attached to the left end of the slide. The upper of the two scales enables the efficiency of dynamos and electric motors to be calculated, or the out-put in kilowatts, or the effective horse-power, with a given degree of efficiency; and this with a single setting of the slide.

By the lower scale, and with only two settings of the slide, the loss of potential in an electric circuit can be calculated from the quantities: current strength, length of lead, and section of conductor. Obviously any one of these factors may be the unknown quantity. The scale is only applicable to direct current calculations, or for an alternating current free from induction.

For simplicity the upper scales will be called the "Efficiency Scale" and the lower graduations, the "Loss of Potential," or "Volt Scale."

The upper scale of the rule is marked "KW," signifying kilowatts, on the right; the upper scale on the slide is marked "H.P.," signifying effective horse-power. The upper scale on the bottom of the rule, that is, the efficiency scale, gives the efficiency of dynamos (from 100 to the left); and the efficiency of motors (from 100 to the right).

Efficiency of Dynamos.

Example 71. Determine the efficiency of a dynamo of 90 KW and 132 H.P.

Set 13·2 on the upper scale of the slide (corresponding to 132 H.P.) under 90 KW on the upper scale of the rule, and read off, at the left end of the slide, 91·3 per cent. on the efficiency scale for dynamos.

From this example it is at once evident, that with a given degree of efficiency any value of the electrical and effective horse-power can be immediately read off with a single setting of the slide.

For this purpose the left end of the slide is set to the degree of efficiency, and the corresponding values are read off on the two upper scales on the rule and slide respectively.

EXAMPLE 72. Efficiency 90 per cent.

Set the left end of the slide to 90 on the "dynamo" scale, and above H.P. = 20, read KW = 13·41; above H.P. = 50, KW = 33·6; above H.P. = 100, KW = 67·1, etc.

Efficiency of Motors.

EXAMPLE 73. Determine the efficiency of a motor of 20 H.P. and 17·1 KW.

Set 2 on the H.P. scale to 17·1 on the KW scale and read off the efficiency (87·3 per cent.) on the "Motor" scale. From this example it is also evident that for a given efficiency, the power in KW can be read immediately above any H.P.

Scale of Loss of Potential—Volt Scale.

The use of the "Volt" scale is as simple as that of the scale last described. The loss of potential in a simple copper lead with direct current, or with alternating current, free from induction, is determined by the formula $e = \frac{J \times L}{c \times q}$, in which e denotes the loss of potential in volts, J, the strength of current in amperes, L the single length of lead in metres, q, the necessary sectional area of the conductor, and c a copper constant, which on the slide rule has been taken as 28·7. The "Volt" scale gives the loss of potential from 0·5 to 10 directly in volts.

EXAMPLE 72. Determine the loss of potential for a copper lead of 70 sq. mm. section and 80 metres in length, with a current strength of 60 amperes.

Set 1 on the upper scale on the slide to 6 on the upper scale on the rule, bring the cursor line to 8 on the upper scale on the slide (obtaining the product J × E); then bring 7 on the upper scale on the slide to the cursor line (obtaining the quotient $\frac{J \times L}{q}$) and read, at the left end of the slide of the "Volt" scale, 2·38 volts. The advantage afforded by the slide rule now becomes evident, for if the loss of

potential found is too large, a further setting of the slide enables the sectional area to be found for any desired loss of potential. Suppose, for example, the loss of potential is only to be 1 volt. The cursor is kept unchanged in position, the left end of the slide set to 1 on the " Volt " scale and 167 sq. mm. is read off under the line on the cursor on the upper scale of the slide.

If the left end of the slide is set to a certain loss of potential and the cursor to a certain section, then by moving the slide, all values of the product " length of lead " and " strength of current " can be obtained, which are applicable to the known section and loss of potential.

Thus by means of the " Volt " scale, any one of the factors in the above formula can be very easily ascertained if the other factors are known.

The rule has the constants 28·7 and 746 marked on the upper scales of both rule and slide, while on the back is a collection of useful constants and other data.

INSTRUCTIONS

FOR THE

"Pickworth" Slide Rule.

The special feature of the "Pickworth" Slide Rule is the provision of an additional scale in the back of the rule. This scale, which we shall designate the "F" Scale, is a single scale one-third the length of the C or D Scales. Used in conjunction with the latter the new scale enables cubes and cube roots to be determined in an exceedingly simple and convenient manner.

On the C and D Scales are Indicators or Gauge Points marked I., II. and III., the two latter being placed at one-third and two-thirds of the length of the lower scales. On the back of the slide is a small index mark which, seen through the slit in the back of the stock of the rule, can be set to any graduation of the F Scale.

In using the rule for Cubes and Cube Roots, the slide requires to be moved *to the right only, such movement never being more than one-third the length of the slide.* In other words, I. on C will always lie between I. on D and the Indicator II. on D. With this limited movement, cubes and cube roots can be determined without hesitancy as to the scales to use and the values to be read, as is inevitable when the operations are carried out with the ordinary scales.

To find the Cube of a Number.

The marks II. and III. on D divide that scale into three equal parts. If the number to be cubed is in the first section, set I. on C to it; if in the second section, set II. to it; if in the third section, set III. to it. Under the index line on the back of the slide read the significant figures of the cube on scale F. If I. on C was used for the setting, the cube contains 1 digit; if II. was used, 2 digits; if III. was used, 3 digits.

EXAMPLE. $1{\cdot}67^3 = 4{\cdot}65$.

Set I. on C to 167 on D and read 465 on F as the significant figures of the cube. Since Indicator I. was used for the setting, there will be one digit in the cube, and $1{\cdot}67^3 = 4{\cdot}65$.

EXAMPLE. $23{\cdot}4^3 = 12{\cdot}8$.

Indicator II. is used and the significant figures read on F are 128. Hence there are 2 digits in the cube and $2{\cdot}34^3 = 12{\cdot}8$.

EXAMPLE. $5{\cdot}77^3 = 192$.

Indicator III. is used; hence there are 3 digits in the cube. The significant figures on F are 192; therefore, $5{\cdot}77^3 = 192$.

In these examples, the first figure of the number to be cubed is in the *units* place. For numbers of any other form, the decimal point is moved through n places so as to bring the first significant figure in the units place, the cube found as above, and the decimal point moved in the *reverse* direction through $3n$ places.

EXAMPLE. $264^3 = 18{,}400{,}000$.

Treat as $2{\cdot}64$, and find $2{\cdot}64^3 = 18{\cdot}4$. As the point is moved 2 places to the *left*, the point in the result is moved $3 \times 2 = 6$ places to the *right* and $264^3 = 18{,}400{,}000$.

EXAMPLE $0{\cdot}0286^3 = 0{\cdot}0000234$.

To bring the first significant figure to the units place requires a movement of $n = 2$ places to the *right*. Then $2{\cdot}86^3 = 23{\cdot}4$; and moving the decimal point $3n = 3 \times 2 = 6$ places to the *left*, we have $0{\cdot}0286^3 = 0{\cdot}0000234$.

Cube Roots.

TO FIND THE CUBE ROOT OF A NUMBER.—*Set the index mark on the back of the slide to the significant figures of the number on scale F and read the cube root on D under I., II. or III. on C, according as the number has 1, 2 or 3 digits preceding the decimal point.*

Thus, setting to 8 on scale F, we have:—

$$\text{Under I. on C } \sqrt[3]{8} = 2.$$

$$\text{,, II. on C } \sqrt[3]{80} = 4{\cdot}31.$$

$$\text{,, III. on C } \sqrt[3]{800} = 9{\cdot}3.$$

Thus under the three Indicators (I., II. and III.) on C, we can always read $\sqrt[3]{N}$; $\sqrt[3]{N \times 10}$; $\sqrt[3]{N \times 100}$, on D, N being less than 10 and not less than 1.

EXAMPLE. $\sqrt[3]{3{\cdot}65} = 1{\cdot}54$.

Set the index mark to 365 on F, and as the number has 1 figure before the decimal point, read the root, 1·54, on D, under I. on C.

Example. $\sqrt[3]{29{\cdot}5} = 3{\cdot}09.$

Set index to 295 on F and read 3·09 on D under II. on C.

Example. $\sqrt[3]{705} = 8{\cdot}9.$

Set index to 705 on F and read 8·9 on D under III. on C.

The examples given have either 1, 2 or 3 figures preceding the decimal point. Numbers of any other form are brought to one of the above forms by moving the decimal point 3 places (or such multiple of 3 places as may be required), the root found and its decimal point moved 1 place for each 3-place movement, but in the *opposite* direction.

Example. $\sqrt[3]{1260000} = 108.$

Moving the decimal point through 2 3-place groups to the left, we obtain 1·26, and $\sqrt[3]{1{\cdot}26} = 1{\cdot}08$. Moving the point 2 places to the *right* we have 108 as the root required.

Example. $\sqrt[3]{14000} = 24{\cdot}1.$

Here 1 3-place movement gives 14. Then $\sqrt[3]{14} = 2{\cdot}41$ and moving the point 1 place we have 24·1 as the root required.

Example. $\sqrt[3]{250000} = 63.$

Here 1 3-place movement gives 250; and $\sqrt[3]{250} = 6{\cdot}3$. Moving the point 1 place gives 63 as the root.

Example. $\sqrt[3]{0{\cdot}0000455} = 0{\cdot}0357.$

Here the point is moved to the *right* through 2 3-place groups, giving 45·5. Then $\sqrt[3]{45{\cdot}5} = 3{\cdot}57$; and moving the point 2 places to the *left* we have 0·0357 as the root required.

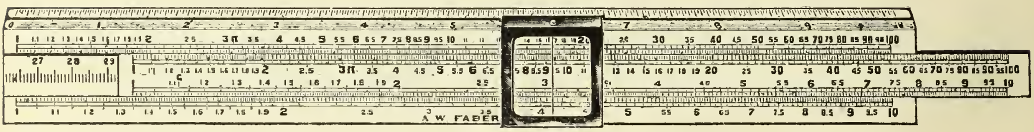
Example. $\sqrt[3]{0{\cdot}32} = 0{\cdot}684.$

Here 1 3-place movement gives 320. $\sqrt[3]{320} = 6{\cdot}84$, and moving the point 1 place to the *left* gives 0·684 as the root required.

BELOW IS A LIST OF THE VARIOUS **CALCULATING RULES**
SUPPLIED BY

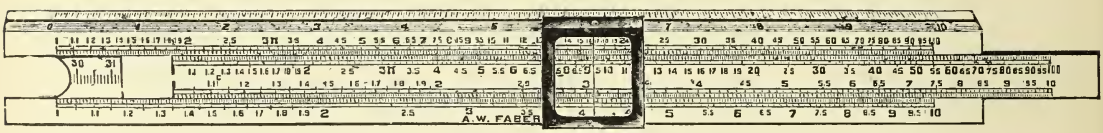
A. W. FABER.

The Rules are made of Boxwood, and have a surface of dull celluloid, the various divisions on which are SCORED or INCISED and marked in BLUE, a great gain in distinctness being thereby obtained. The celluloid surfaces are pinned to the boxwood, to give additional security.

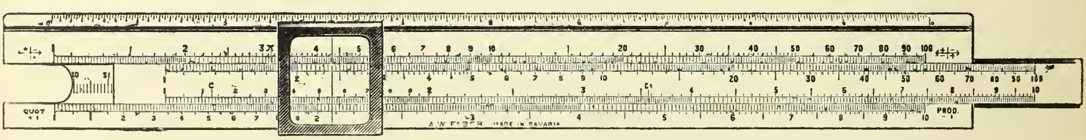


360. REDUCED SIZE.

No. 360. **Calculating Rule**, with decimals *marked*, with **Spring Back and Ordinary Cursor**, $10\frac{1}{4}$ in. long, celluloid surface.



363. REDUCED SIZE.

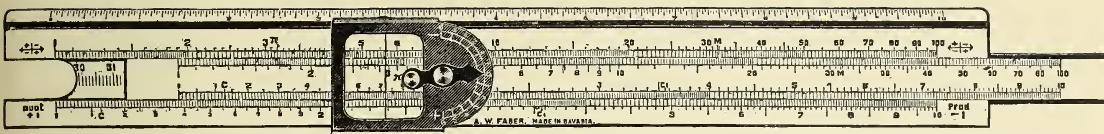


364. REDUCED SIZE.

No. 363. **Calculating Rule**, with decimals *marked*, with **Spring Back and Ordinary Cursor**, 11 in. long, celluloid surface.

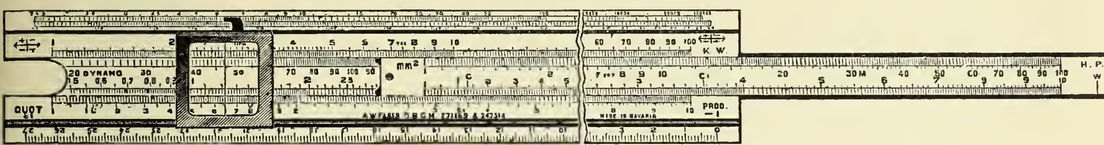
No. $363\frac{1}{2}$. **Calculating Rule**, with decimals *marked*, with **Spring Back and Registering Cursor**, 11 in. long, celluloid surface.

No. 364. **Calculating Rule**, *without* decimals, with **Spring Back and Ordinary Cursor**, 11 in. long, celluloid surface.



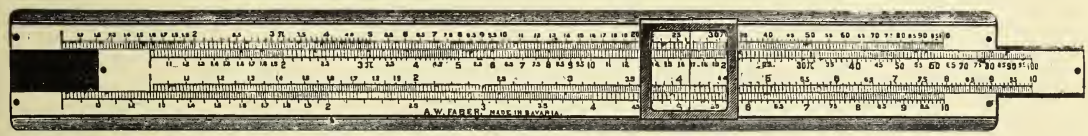
367. REDUCED SIZE.

No. 367. **Calculating Rule,** *without* decimals, with **Spring Back and Registering Cursor,** 11 in. long, celluloid surface.



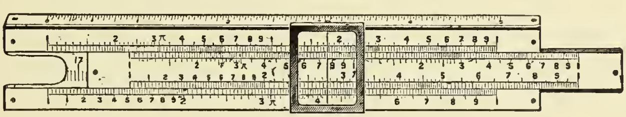
368. REDUCED SIZE.

No. 368. **Calculating Rule,** *without* decimals, with **Log=Log Scale, Spring Back and Special Cursor,** 11 in. long, celluloid surface.



361. REDUCED SIZE.

No. 361. **Calculating Rule,** *with* decimals, for Students' use, two bevelled measuring edges, 11 in. long, celluloid surface.



369. REDUCED SIZE.

No. 369. **Calculating Rule,** *without* decimals, with **Ordinary Cursor,** 6 in. long, celluloid surface.

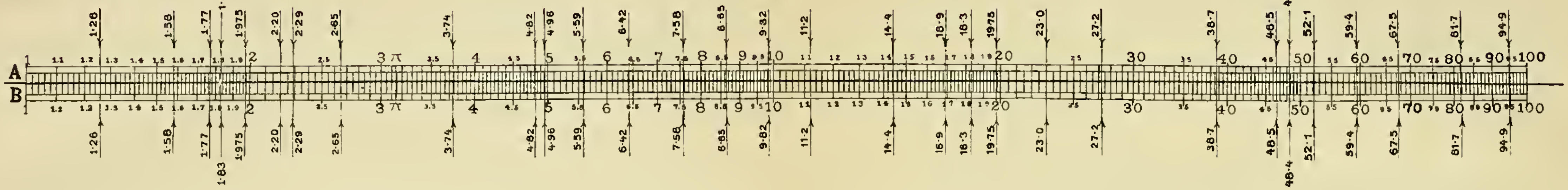
No. 374. **"Pickworth" Calculating Rule,** *without* decimals, with **new "F" Scale for Cubes and Cube Roots, Ordinary Cursor,** 11 in. long, celluloid surface.

No. 377. **"Pickworth" Calculating Rule,** *without* decimals, with **new "F" Scale for Cubes and Cube Roots, Registering Cursor,** 11 in. long, celluloid surface.

Manufactory established 1761.

UPPER SCALE

FIG. 1.



LOWER SCALE

FIG. 2.

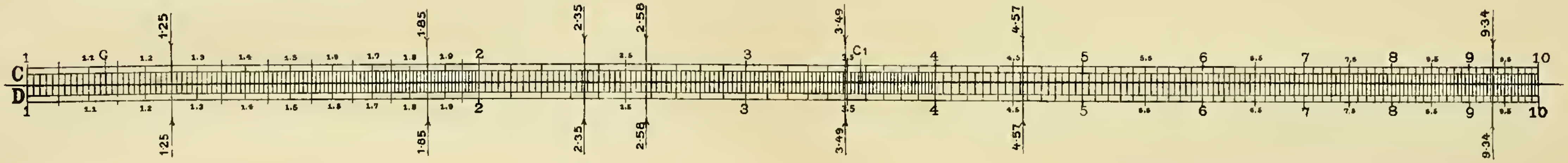


FIG. 3.

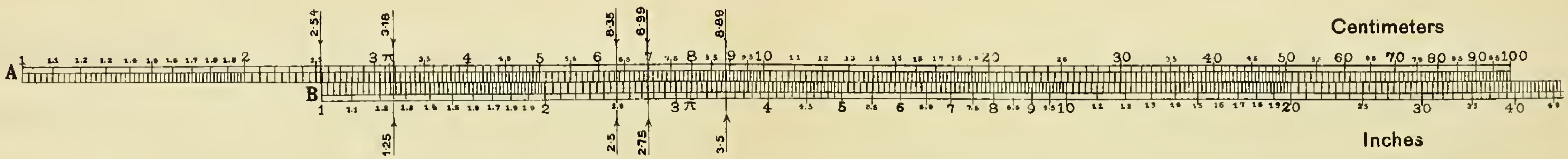


FIG. 4.

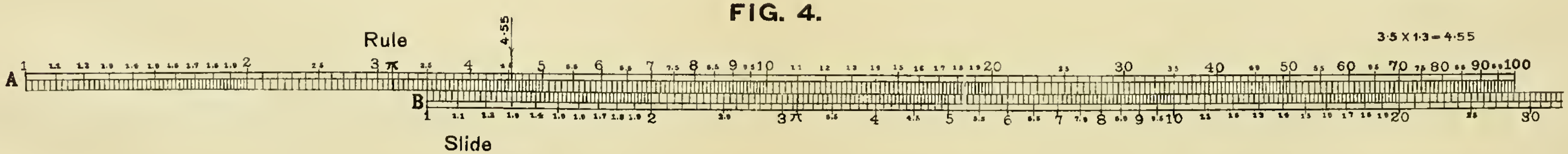


FIG. 5.

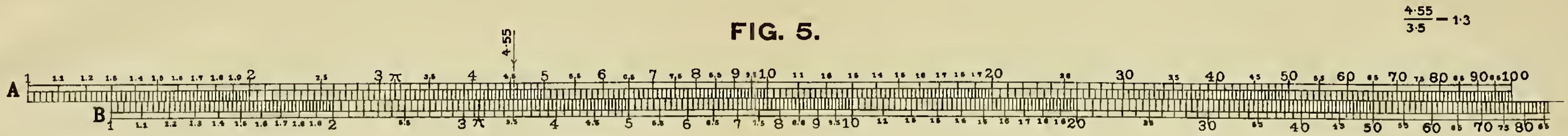


FIG. 6.

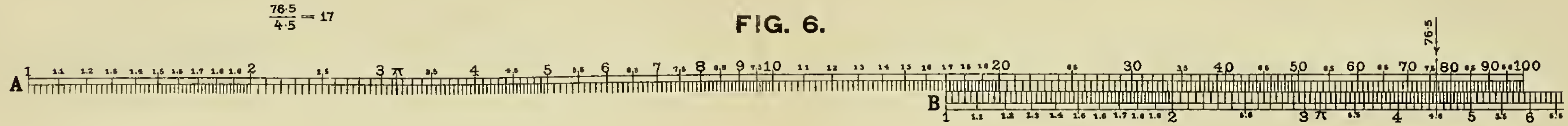


FIG. 7.

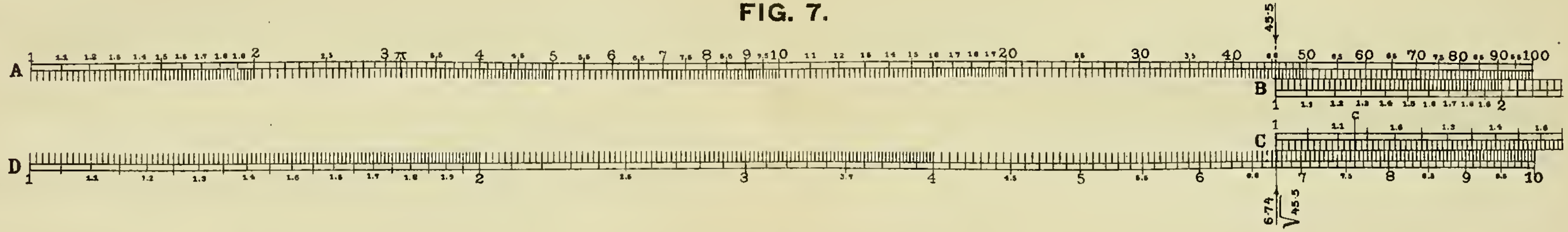


FIG. 8.

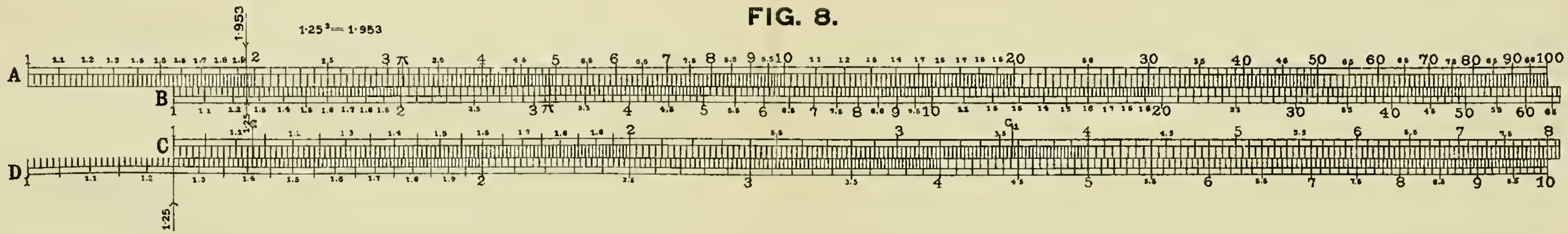


FIG. 9.

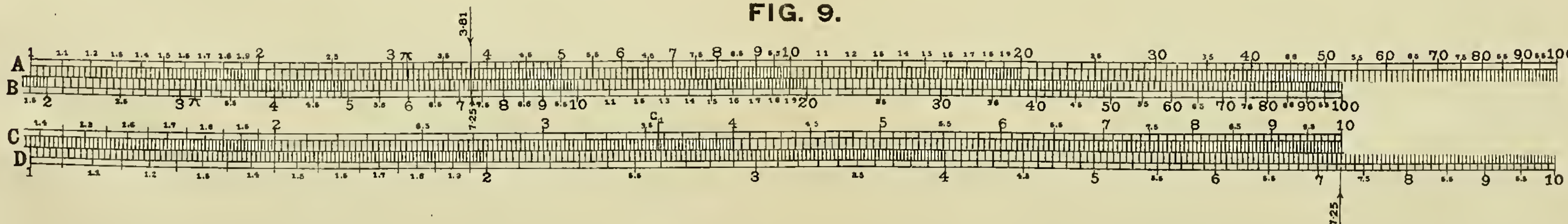


FIG. 10.

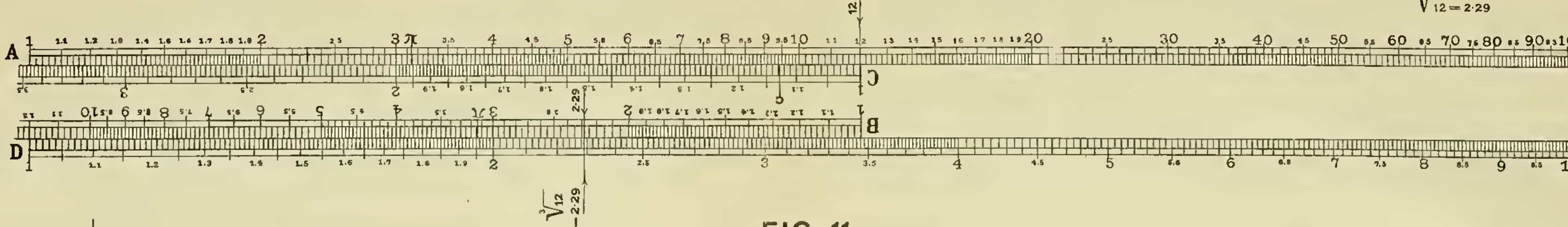


FIG. 11.

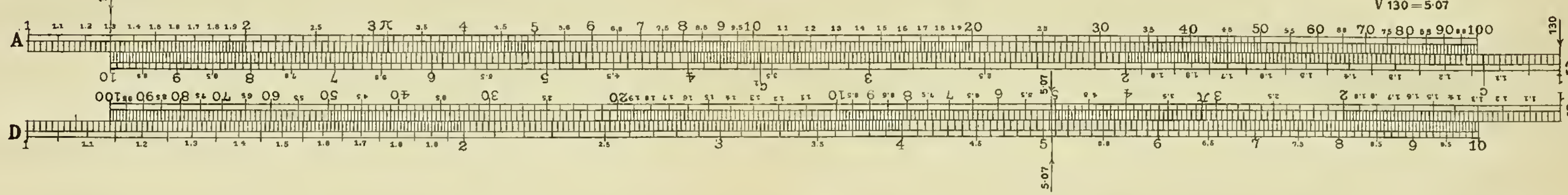


FIG. 12.

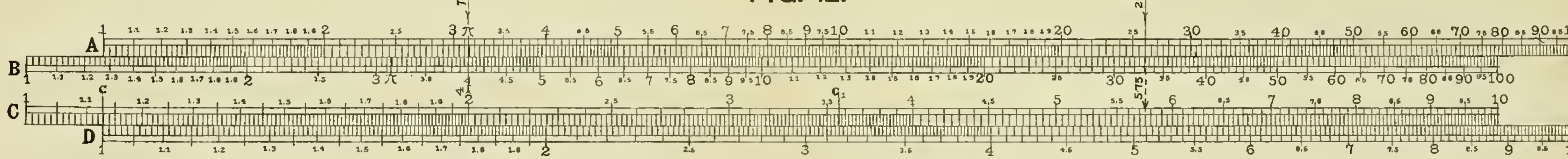


FIG. 13.

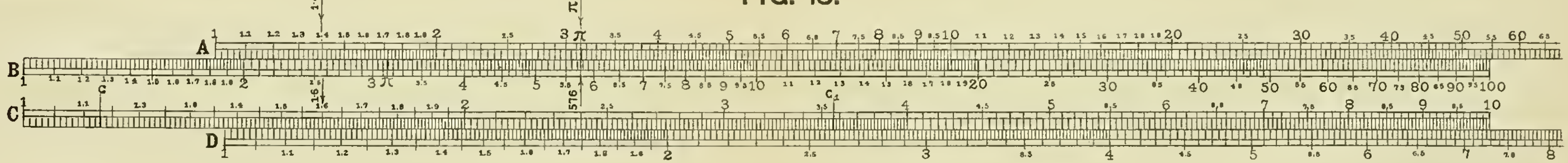


FIG. 14.

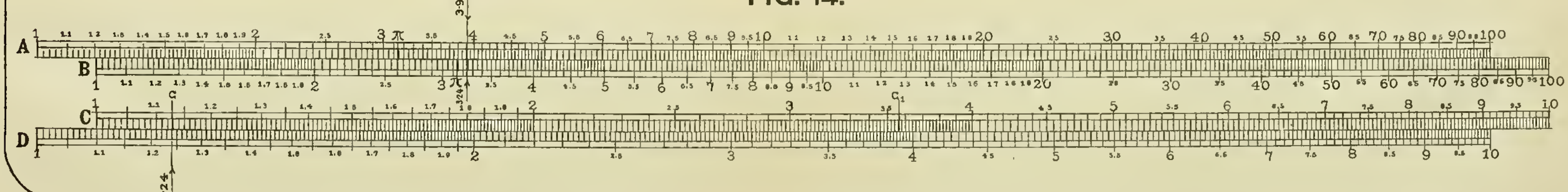


FIG. 15.

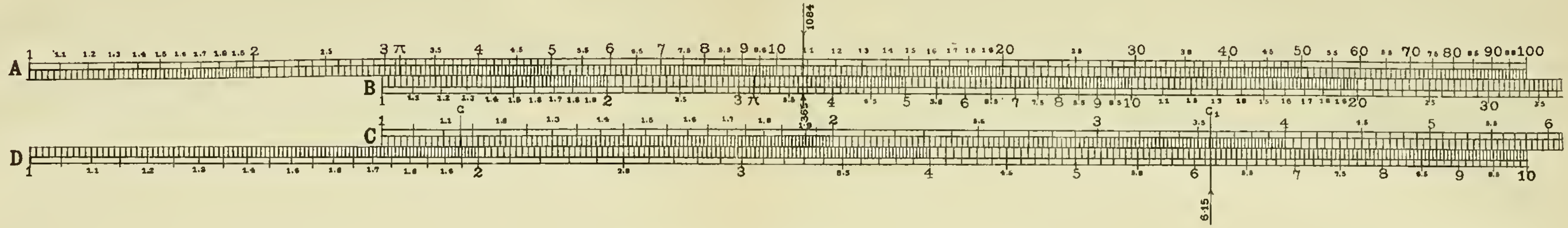


FIG. 16.

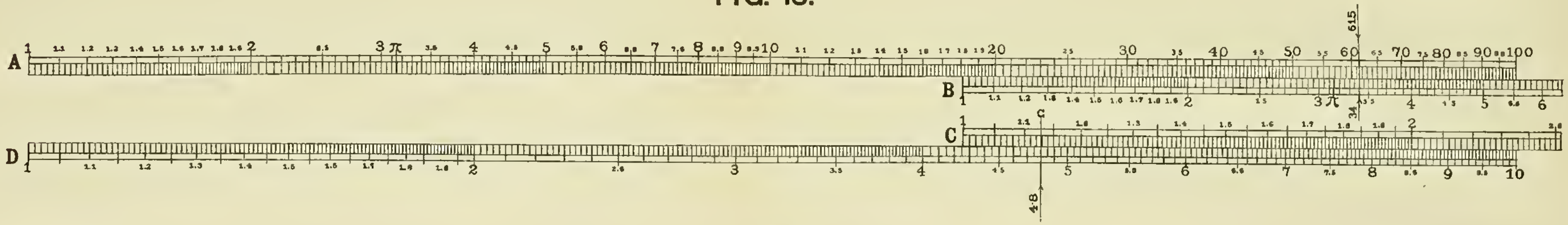


FIG. 17.

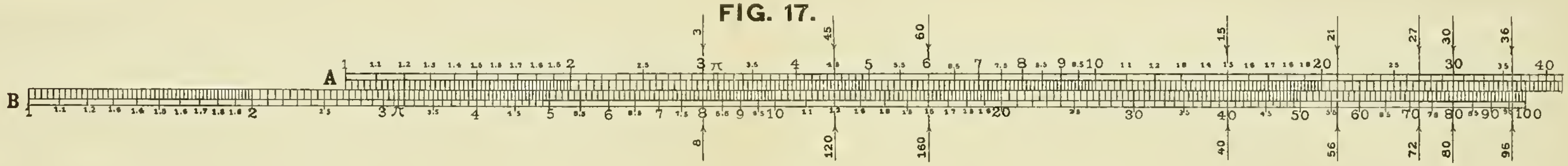


FIG. 18.

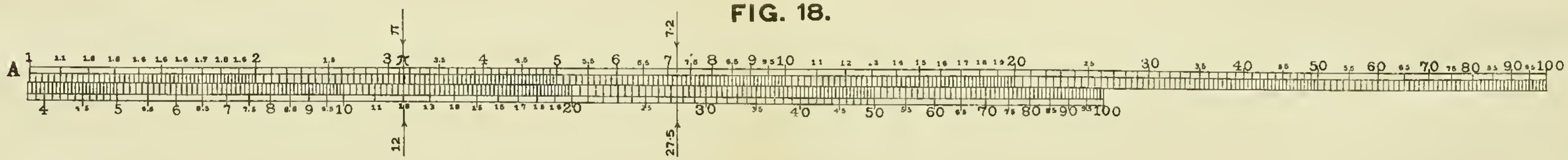


FIG. 19.

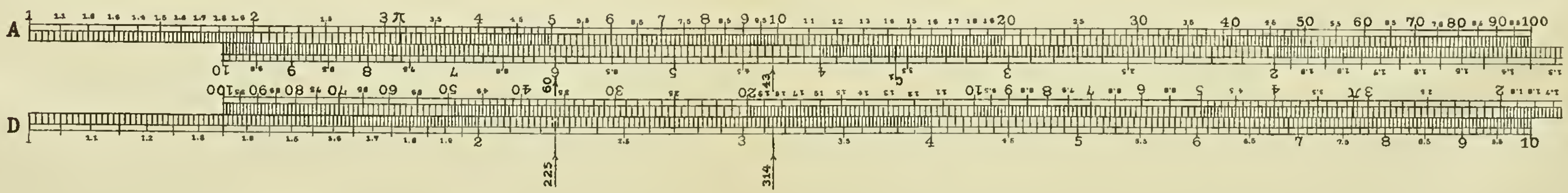


FIG. 20.

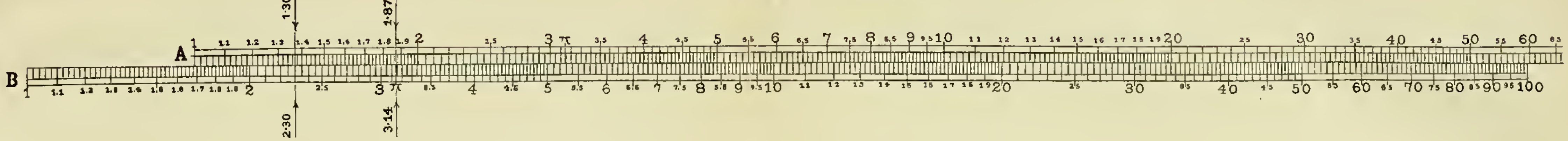


FIG. 21.

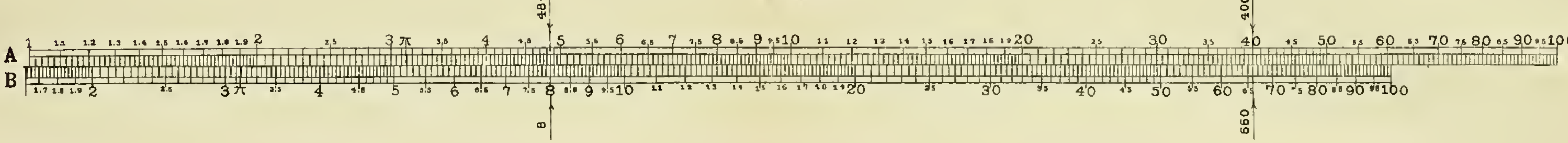


FIG. 22.

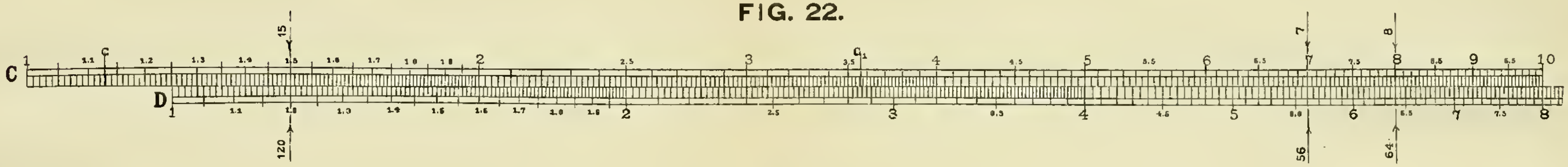


FIG. 23.

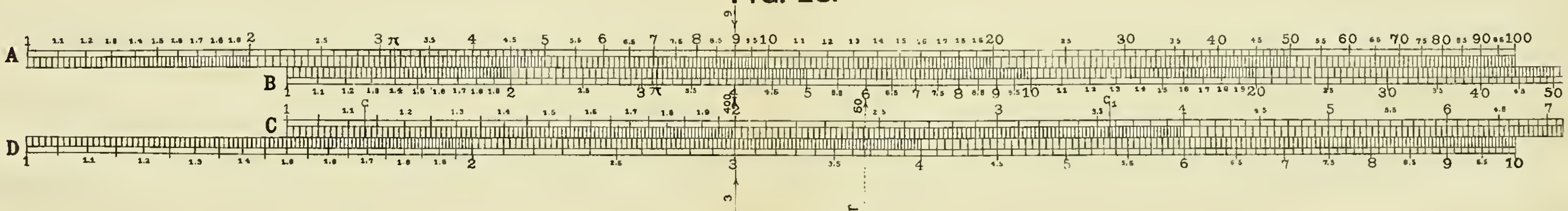


FIG. 24.

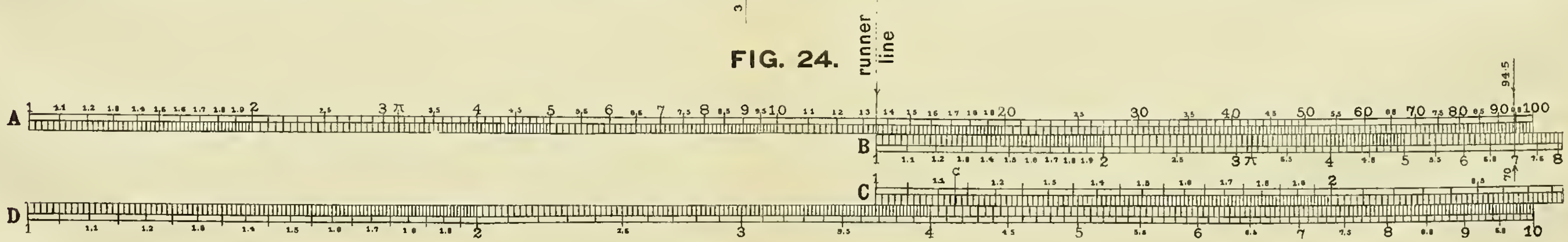


FIG. 25.

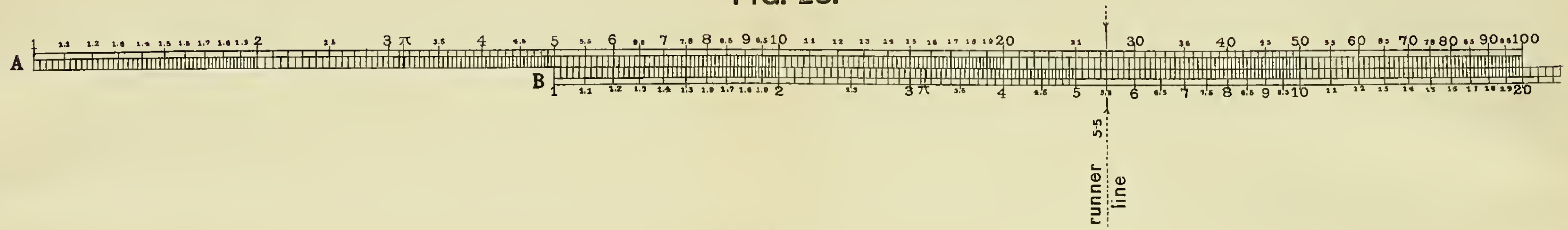


FIG. 26.

FIG. 27.

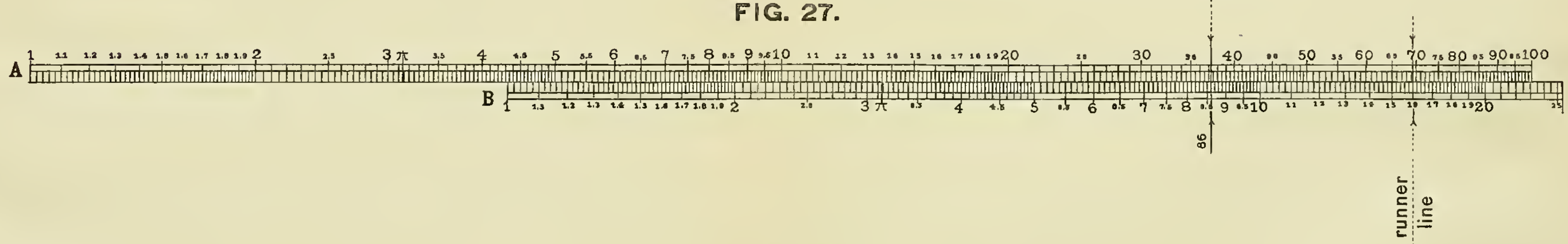


FIG. 28.

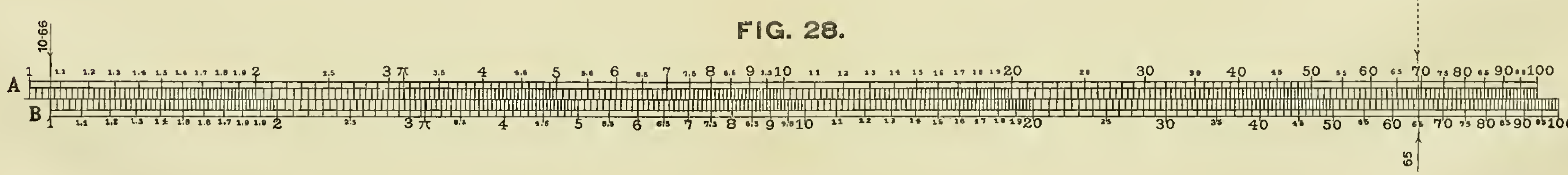


FIG. 29.

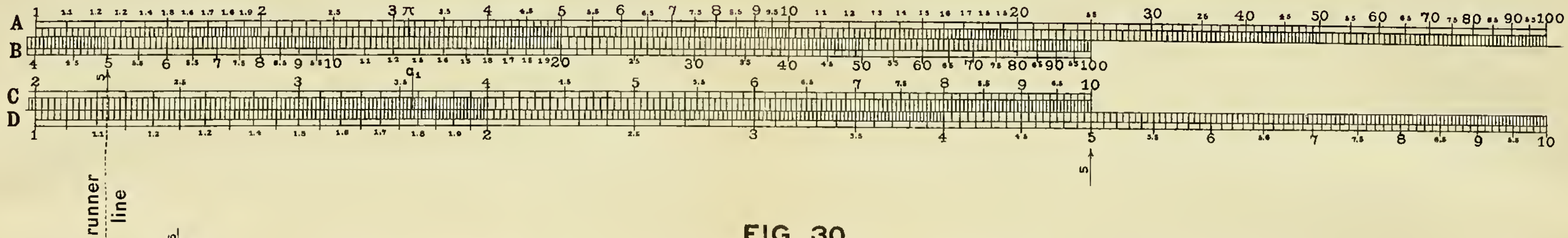


FIG. 30.

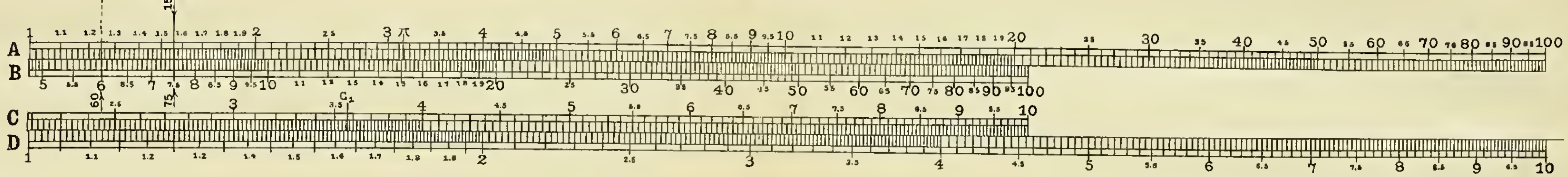


FIG. 31.

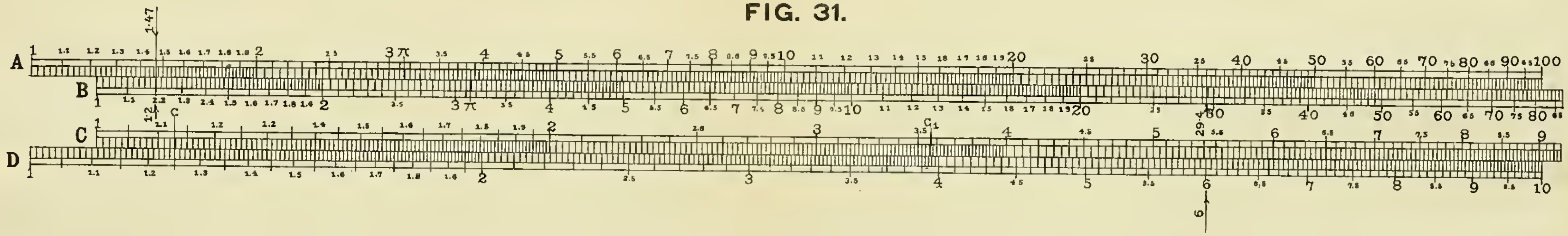


FIG. 32.

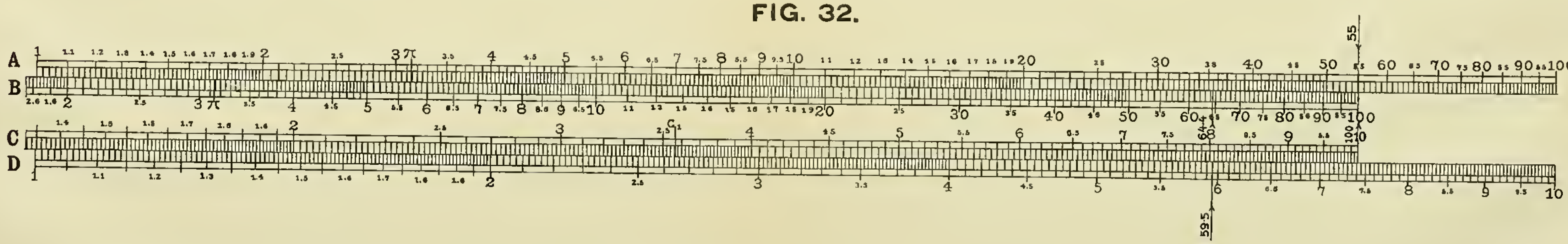


FIG. 33.

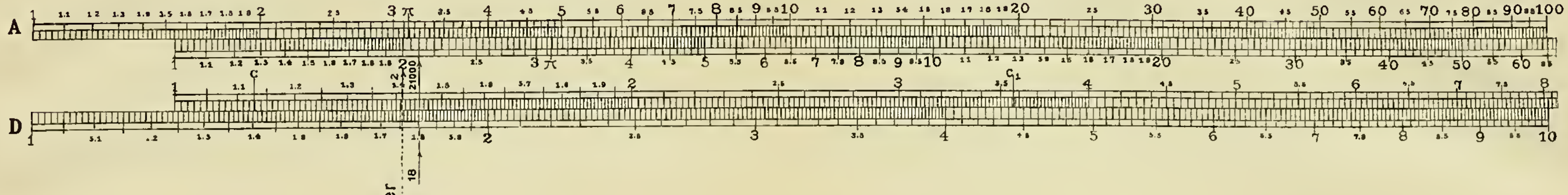


FIG. 34.

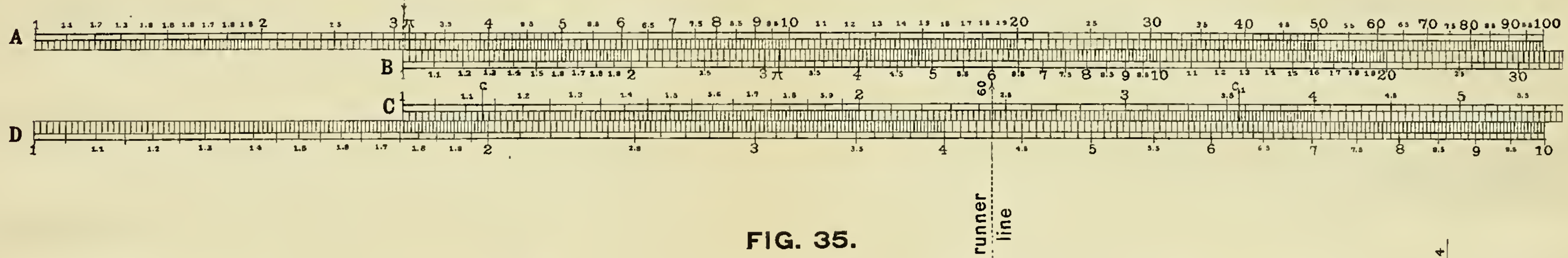


FIG. 35.

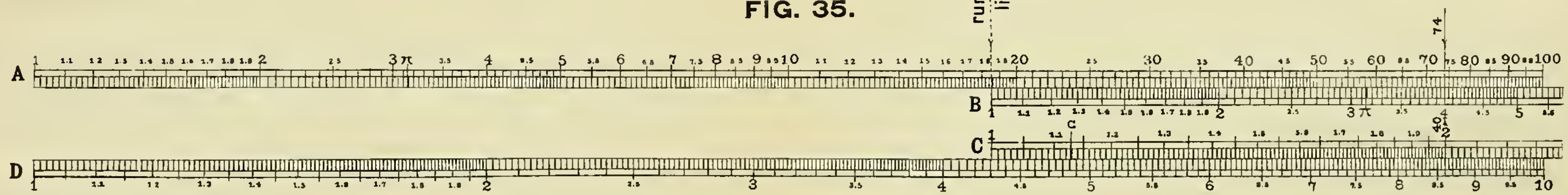


FIG. 36.

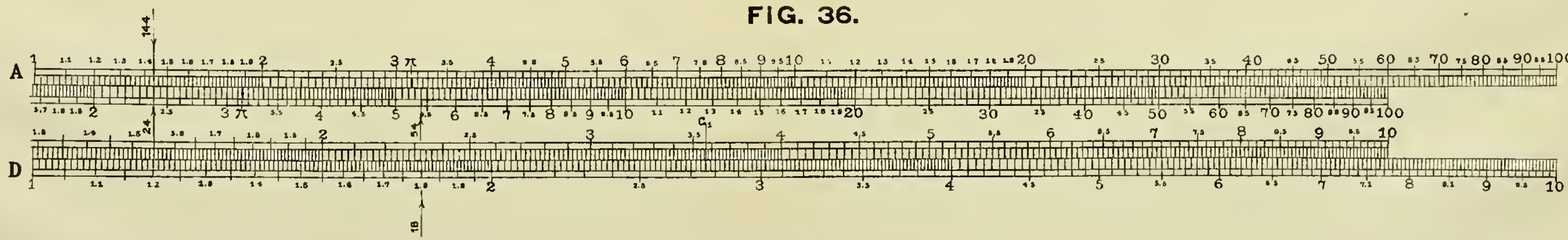


FIG. 37.

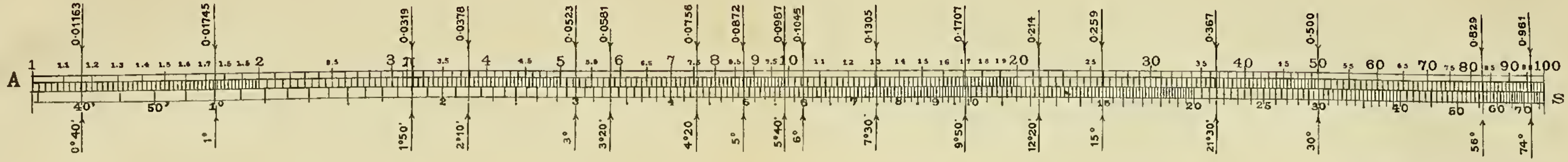


FIG. 38.

FIG. 39.

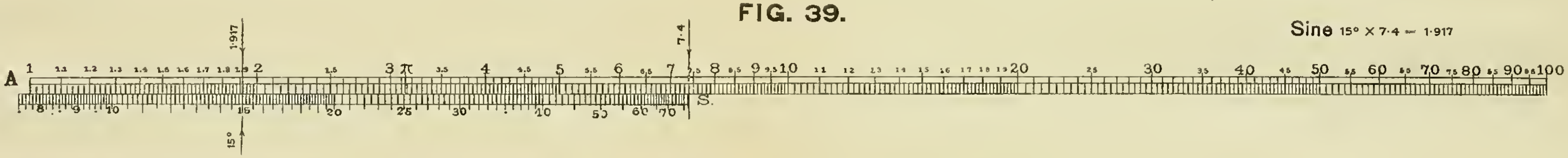


FIG. 40.

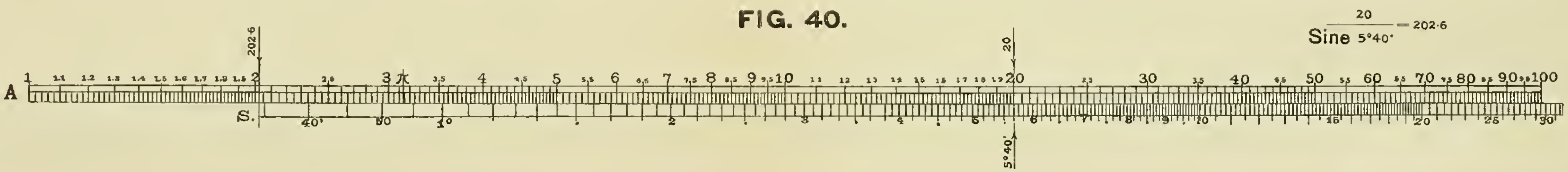
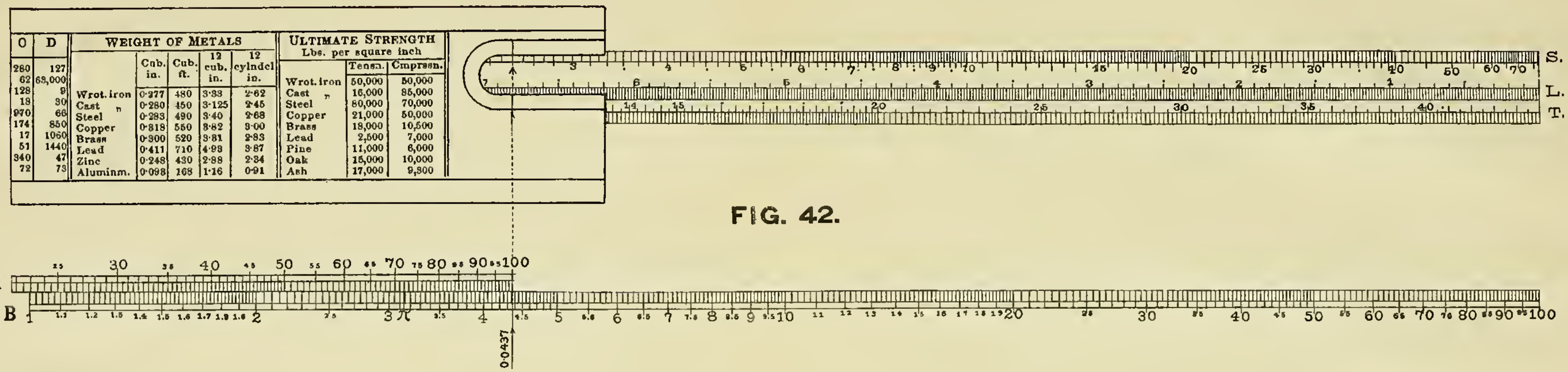


FIG. 41.



O	D	WEIGHT OF METALS	Cub. in.	Cub. ft.	12 cub. in.	12 cylndcl in.	ULTIMATE STRENGTH Lbs. per square inch	Tensn.	Cmpresn.
280	127								
62	63,000						Wrot. iron	50,000	50,000
128	9	Wrot. iron	0·277	480	3·33	2·62	Cast „	16,000	85,000
18	30	Cast „	0·260	450	3·125	2·45	Steel	80,000	70,000
970	66	Steel	0·283	490	3·40	2·68	Copper	21,000	50,000
174	850	Copper	0·318	550	3·82	3·00	Brass	18,000	10,500
17	1060	Brass	0·300	520	3·81	2·83	Lead	2,500	7,000
51	1440	Lead	0·411	710	4·93	3·87	Pine	11,000	6,000
340	47	Zinc	0·248	430	2·88	2·34	Oak	15,000	10,000
72	73	Aluminm.	0·098	168	1·16	0·91	Ash	17,000	9,300

FIG. 42.

FIG. 43.

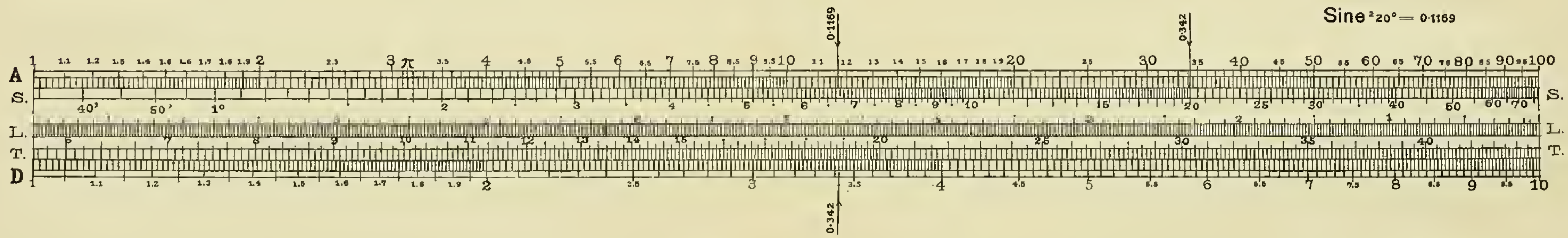


FIG. 44.

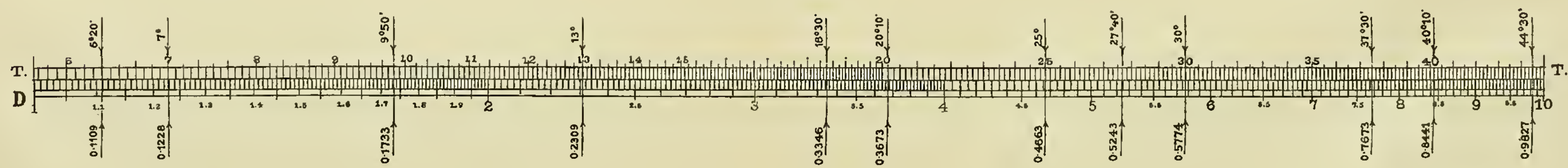


FIG. 45.

$\tan 60° = \frac{1}{\tan 30°}$

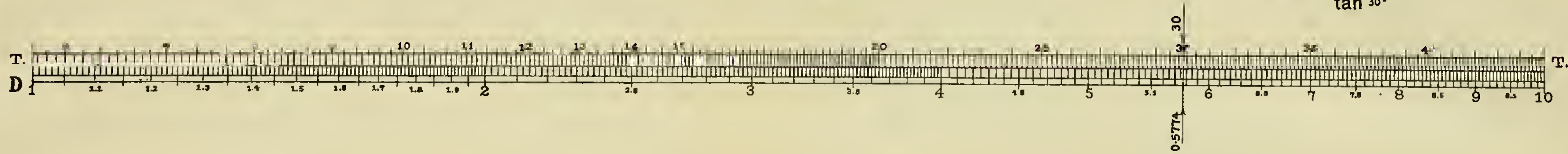
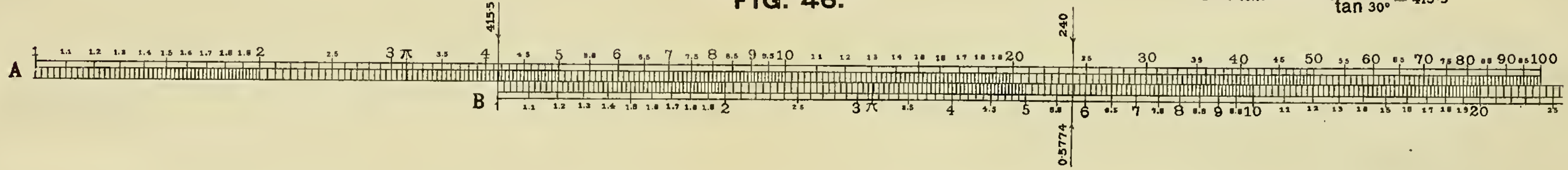


FIG. 46.

$240 \times \tan 60° = \frac{240}{\tan 30°} = 415{\cdot}5$



log. 35·8 = 1·554

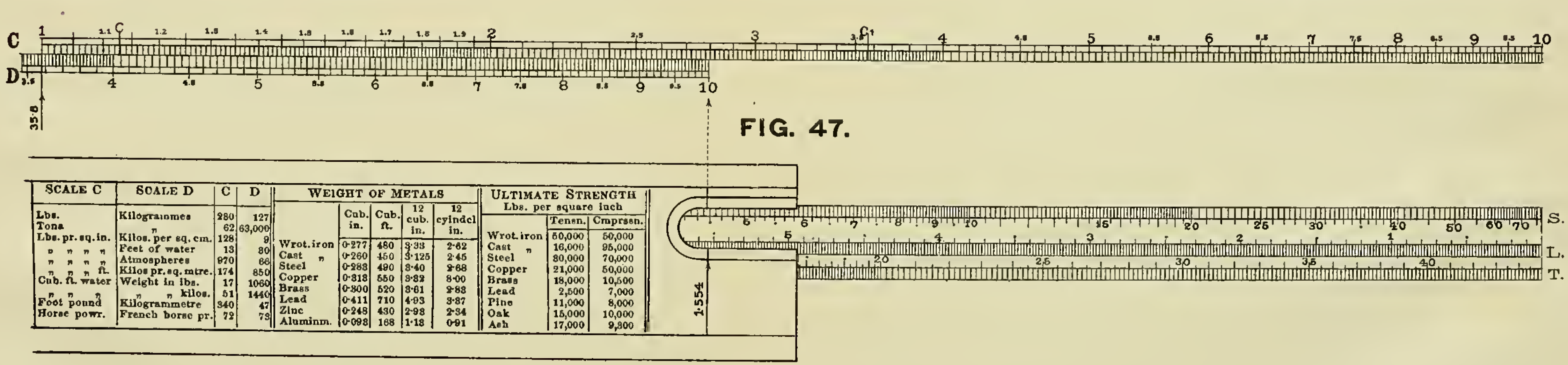


FIG. 47.

SCALE C	SCALE D	C	D
Lbs.	Kilogrammes	280	127
Tons	"	62	63,000
Lbs. pr. sq. in.	Kilos. per sq. cm.	128	9
" " " "	Feet of water	13	30
" " " "	Atmospheres	970	66
" " " ft.	Kilos pr. sq. mtre.	174	850
Cub. ft. water	Weight in lbs.	17	1060
" " "	" " kilos.	51	1440
Foot pound	Kilogrammetre	340	47
Horse powr.	French horse pr.	72	73

WEIGHT OF METALS	Cub. in.	Cub. ft.	12 cub. in.	12 cyindcl in.
Wrot. iron	0·277	480	3·33	2·62
Cast "	0·260	450	3·125	2·45
Steel	0·283	490	3·40	2·68
Copper	0·318	550	3·82	3·00
Brass	0·300	520	3·61	2·83
Lead	0·411	710	4·93	3·87
Zinc	0·248	430	2·98	2·34
Aluminm.	0·098	168	1·18	0·91

ULTIMATE STRENGTH Lbs. per square inch	Tensn.	Cmprssn.
Wrot. iron	50,000	50,000
Cast "	16,000	95,000
Steel	80,000	70,000
Copper	21,000	50,000
Brass	18,000	10,500
Lead	2,500	7,000
Pine	11,000	8,000
Oak	15,000	10,000
Ash	17,000	9,300

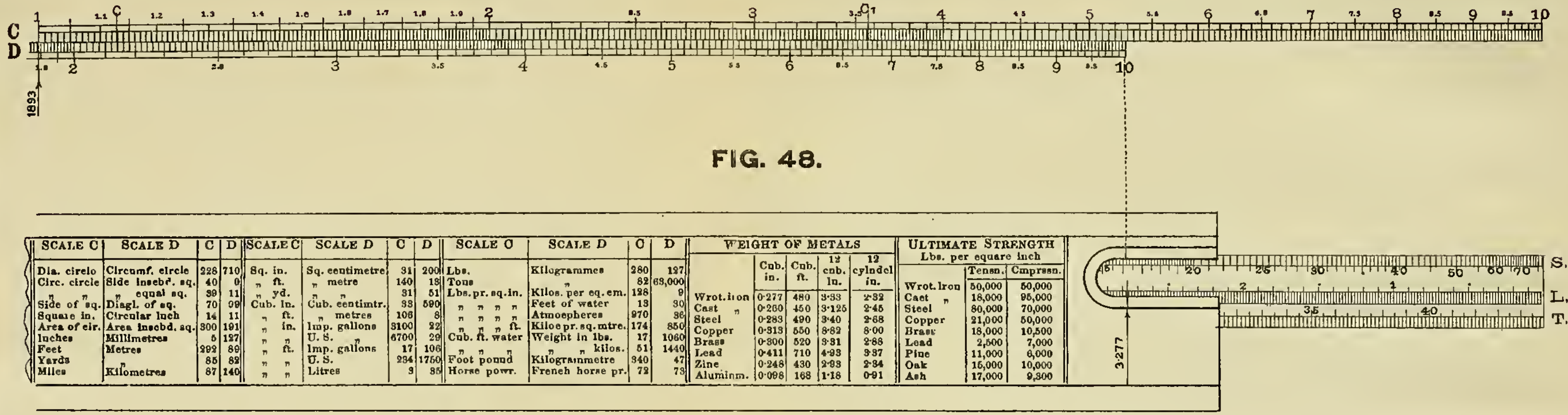


FIG. 48.

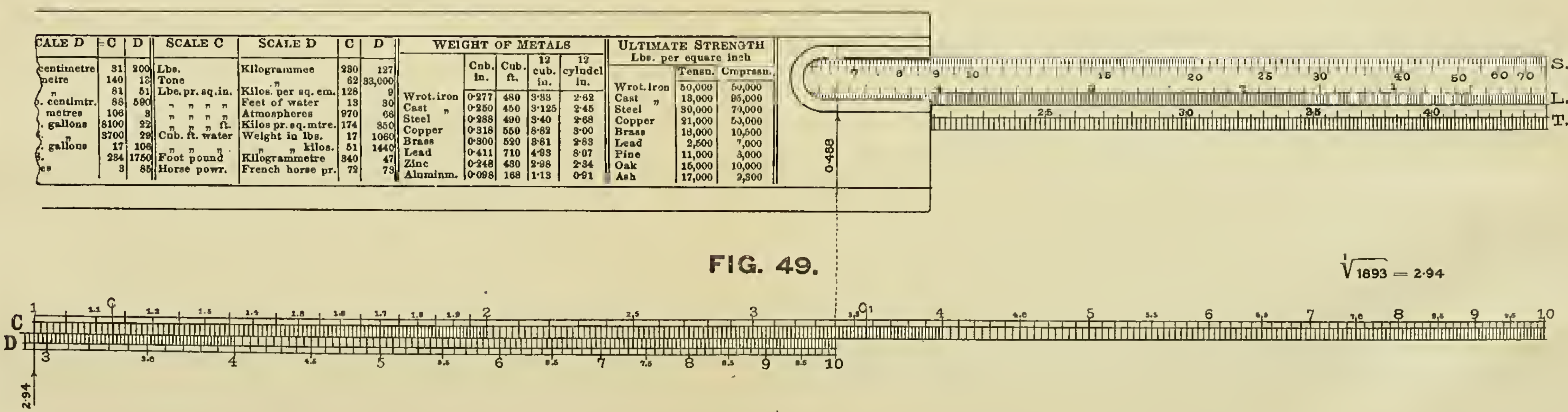


FIG. 49.

$\sqrt[3]{1893} = 2{\cdot}94$

FIG. 50.

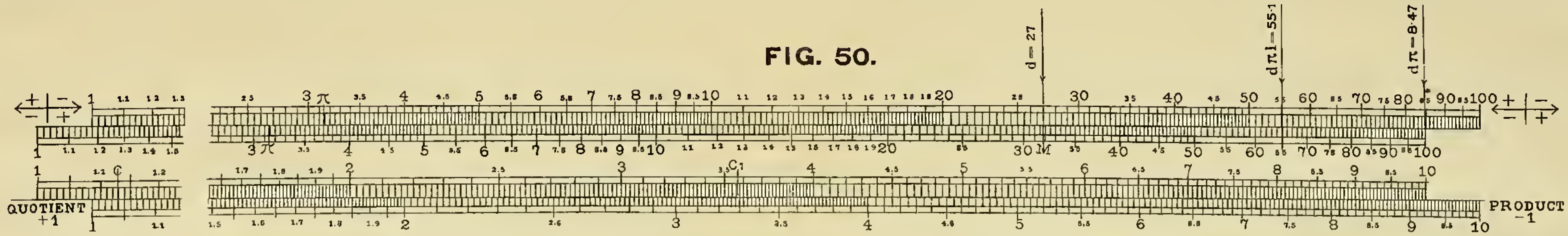


FIG. 51.

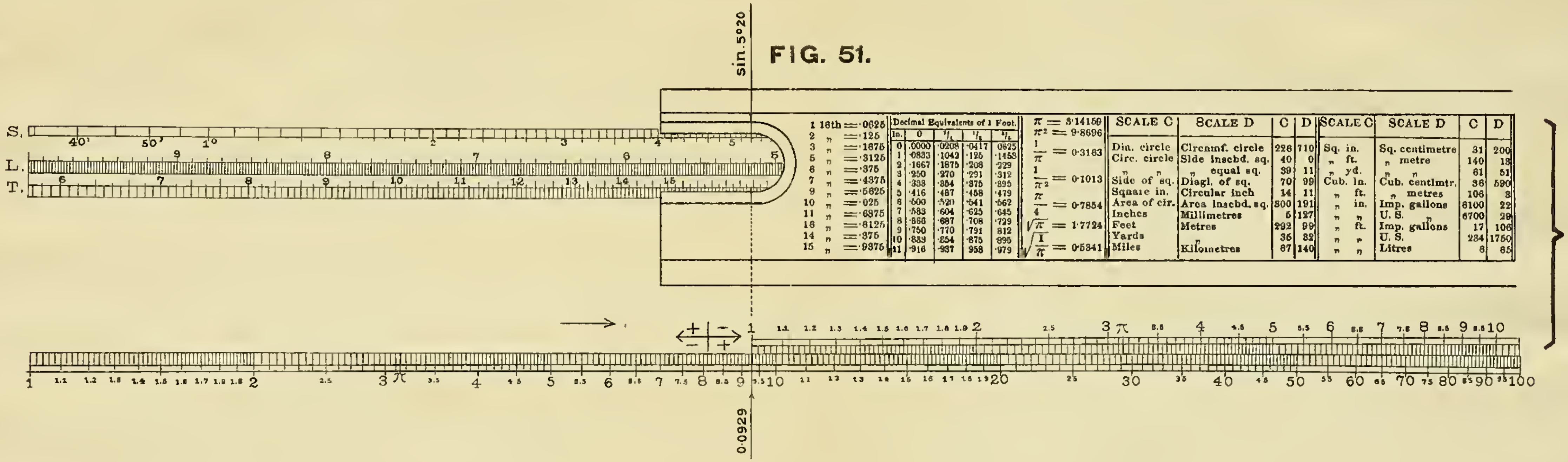


FIG. 52.